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Cracking the JEE main Code
KISHALAY 350 out of 360

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PHYSICS for you

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Contents

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Editor : Anil Ahlawat

editorial

The March towards manufacture of tailor made two dimensional crystals

The Saga of single crystals to crystals of two dimensions is a very exciting one. The facilities for growing a single crystal were available only in a few top laboratories. For spectroscopy studies, these crystals have to grown in their own laboratories. Our laboratories in India were making their own crystals.

The race for making thin crystals culminated in the discovery of graphene. This is a two dimensional sheet of carbon atoms. For the semiconductors, the band width is an important property. This is essential for making computer chips and solar cells.

Research scientists at Massachusetts Institute of Technology and Harward University have discovered a two dimensional material similar to grapheme with a band width that is manageable.

The new material is a combination of nickel and an organic compound called HITP also has the advantage of self-assembly. Its constituents naturally adjust-themselves. This could make manufacturing simple. By adjusting relative amounts of the materials, desired properties of band widths could be obtained.

The new compound has the same structure as graphene that is to say a perfectly hexagonal honey comb structure. For our future scientists, the field of conductors, semi-conductors and insulators are still fertile.

The conductivity of non-conductors still remains a hot topic for research.

Anil Ahlawat
Editor

Cracking the JEE Main CODE

Kishalay Raj

JEE Main 2014 Topper Interview : How Kishalay Raj scored 350 out of 360?

In this interview, Kishalay says his focus has been JEE Advanced 2014 since the very early stages of JEE preparations and as a result he did not find JEE Main 2014 paper any difficult. His father Rajesh Srivastava who works with Tata Steel in Jamshedpur (Jharkhand) and mother Kshama Srivastava who is a homemaker, say that his son has been self-disciplined in his studies and they did never need to guide him on his routine study schedules.

Congratulations Kishalay for scoring 2^{nd} highest in JEE Main 2014.

Thank you!

What is your overall and subject wise score in JEE Main 2014? What was your class X marks?

My overall score is 350. I scored 120 marks in Mathematics and 115 in each Physics and Chemistry in JEE Main 2014. I had around 98% in my class X from ICSE board.

How did you prepare for JEE Main 2014?

I did not specifically prepare for JEE Main 2014. Since the beginning of my preparation, I was focused on JEE Advanced only. This is because syllabus for JEE Main is same as JEE Advanced. By covering all the topics under three subjects for JEE Advanced, I automatically covered the entire JEE Main syllabus.

Do you mean that preparation for JEE Advanced since the very early stages of your preparation made you score high in JEE Main as well?

Yes. It definitely did. I was confident that JEE Advanced preparation will get me through JEE Main automatically. Still, I was expecting around 320-330, but did not expect this high score.

How much time did you devote to your studies during JEE preparations?

I used to study about 8-9 hours a day during my JEE preparation phase. I would go for regular classes in Prerna Institute in Jamshedpur for 3-4 hours and rest of the time I regularly spent on self study at my home. I avoided skipping any topic under JEE and my Class XII syllabus. The consistency in my preparation boosted my confidence for the paper.

How did you divide your preparation schedule among the three subjects of JEE Main? What are your strong and weak areas?

I divided my time equally among the three subjects. Though Physics is my strongest subject, but I put equal emphasis on all the three. My strong area is my ability to approach quantitative part of all subjects like Mechanics portion in Physics. But, I find qualitative analysis in Chemistry a bit difficult.

Besides studies, what do you do? Do you like sports?

I play computer games whenever I get free time. I like scoring highest in each of my new game! In outdoor games,

I prefer football. Sometimes, I watch Hollywood movies also.

Which is your favourite Hollywood movie? Which movie did you watch last time?

My favourite movie is 'Inception'. I watched 'Iron Man' about two months back.

Once you crack JEE Advanced, which Engineering branch would you like to opt for?

I would like to go for Computer Science Engineering in IIT Bombay.

Why do you want to study Computer Science Engineering?

I like playing with computers. Also, my ideal Sachin Bansal and Binny Bansal who founded Flipkart and made it the biggest online shopping platform, are Computer Science Engineers. I want to follow their footsteps and establish my own venture after I complete my Engineering from IIT.

All the best for your JEE Advanced and dream branch in IIT!

Thank you.

Courtesy : careers360.com

Over 26,000 students who qualified for IIT examination give it a miss

Despite the hype over IITs, more than 26,000 students who were shortlisted for the JEE (advanced) this year, have not registered for the exam for admission to the 16 IITs, registrations.

According to sources, of the top 1,53,981 students who had been selected from the JEE (main), 1,26,990 have registered for the JEE(advanced) exam held on May 25. Around the same number of students had not registered for the exam last year as well.

"There are many students who do not want to be away from their home and parents. There are some states which are no more conducting their own engineering exam. In such a case, the child prefers to take admission in a state engineering college on the basis of his rank in JEE(main)," said HC Gupta, JEE (advanced) 2013 chairperson.

Eight students from Nepal and Bhutan had also qualified for the exam this year. It is not sure whether they have registered or not.

Students from Andhra Pradesh, UP and Rajasthan accounted for nearly 38% of those who have qualified for JEE(advanced). More than 12 lakh students had appeared for JEE(main).

Those who could not make it to JEE(advanced) will seek admission to NITs and other centrally funded institutes according to their final score, which will be known on July 7 after all the board results are out.

Growing number of Indian students now heading to France

Over **800 programmes** are in English, and the faculty's International

French govt. offers **300 scholarships** to Indians each year. Monthly stipends over accommodation, visa fee waiver; and in public institutions, the entire tuition fee is waived. Even private institutions are subsidized. Exchange students are also entitled to scholarships such as Charpak.

University students are entitled to **Caisses d'Allocations Familiales or CAF** – money allocated to students by the French govt. for accommodation

Internships take students to other European countries and travel is generally cheap.

Students are allowed to **stay on and work for a year** after completing the programme but work permits are hard to come by

Those who have done Master's and PhD courses get **5 yr** multiple entry visas

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

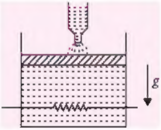
By : Akhil Tewari

PROBLEM Set 11

1. An ideal gas is expanded so that amount of heat given is equal to the decrease in internal energy. The adiabatic exponent if the gas undergoes the process $TV^{1/5}$ = constant, is
   (a) 7/5 (b) 6/5
   (c) 8/5 (d) none of these

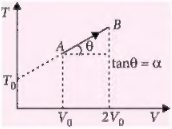
2. Three travelling waves in same direction are superimposed. The equations of wave are $y_1 = A_0 \sin(kx - \omega t)$, $y_2 = 3\sqrt{2}A_0 \sin(kx - \omega t + \phi)$ and $y_3 = 4A_0 \cos(kx - \omega t)$. If $0 \le \phi \le \pi/2$ and the phase difference between resultant wave and first wave is $\pi/4$, then ϕ is
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
   (c) $\frac{\pi}{12}$ (d) none of these

3. A resistance coil, connected to an external battery, is placed inside the thermally insulating smooth piston and containing an ideal monoatomic gas. A current is passed through the coil such that it is generating heat at the rate 30 J s^{-1} inside the cylinder. Volume of the chamber of the cylinder is 5 m^3 and area of the piston is 0.1 m^2. The mass rate of dropping sand on the piston such that the piston always remains stationary will be (Assume that the sand is dropped very swiftly. (Take g = 10 m s^{-2})



   (a) 0.01 kg s^{-1}
   (b) 0.02 kg s^{-1}
   (c) 0.03 kg s^{-1}
   (d) 0.04 kg s^{-1}

4. One mole of an ideal monoatomic gas undergoes the process AB as shown in the T-V indicator diagram. If volume of the system changes from V_0 to $2V_0$, then the amount of heat transferred to the system is

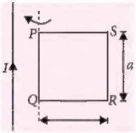


   (a) $\frac{5RV_0}{2} + \frac{RT_0}{\alpha}\ln 2$ (b) $\frac{3RV_0}{2} + \frac{RT_0}{\alpha}\ln 2$
   (c) $\frac{5R\alpha V_0}{2} + RT_0 \ln 2$ (d) $\frac{3R\alpha V_0}{2} + RT_0 \ln 2$

5. A square frame of side a and a long straight wire carrying a current I are located in the same plane. The frame is rotated through an angle of 90° about side PQ. The amount of charge flown through the frame, if resistance of the frame is R, is

(a) $\dfrac{\mu_0 Ia\ln\sqrt{2}}{2\pi R}$

(b) $\dfrac{\mu_0 Ia\ln\sqrt{2}}{\pi R}$



(c) $\dfrac{2\mu_0 Ia\ln\sqrt{2}}{\pi R}$ (d) $\dfrac{\mu_0 Ia\ln(1/\sqrt{2})}{\pi R}$

6. A force $\vec{F}=(-y\hat{i}+x\hat{j})$ N acts on a particle as it moves in anticlockwise circular motion in x-y plane. The centre of the circle is at the origin. If the work done by the force is 32π J in one complete revolution then assuming x, y to be in meters, the radius of the path is
(a) 2 m (b) 4 m
(c) 8 m (d) 16 m

7. The period of oscillation of a simple pendulum is $T=2\pi\sqrt{\dfrac{L}{g}}$. L is about 10 cm and is known to 1 mm accuracy. The period of oscillation is about 0.5 s. The time of 100 oscillation is measured with a wrist watch of 1 s resolution. The percentage error in the determination of g will be
(a) 2% (b) 5%
(c) 6% (d) 8%

8. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between resultant intensities at A and B is
(a) $2I$ (b) $4I$
(c) $5I$ (d) $7I$

9. A lens has a power of +5 D in air. What will be its power if it is completely immersed in water ?
Given $\left(\text{Given } \mu_g=\dfrac{3}{2}; \mu_w=\dfrac{4}{3}\right)$.
(a) $\dfrac{5}{3}$ (b) $\dfrac{3}{2}$
(c) $\dfrac{1}{2}$ (d) $\dfrac{8}{9}$

10. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by
(a) 12 (b) 18
(c) 24 (d) 30

SOLVED PAPER 2014

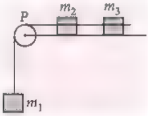
AIPMT

1. If force (F), velocity (V) and time (T) are taken as fundamental units, then the dimensions of mass are
(a) $[FVT^{-1}]$ (b) $[FVT^{-2}]$
(c) $[FV^{-1}T^{-1}]$ (d) $[FV^{-1}T]$

2. A projectile is fired from the surface of the earth with a velocity of 5 m s^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 m s^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in m s^{-2}) is
(Given $g = 9.8$ m s^{-2})
(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8

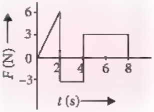
3. A particle is moving such that its position coordinates (x, y) are
(2 m, 3 m) at time $t = 0$,
(6 m, 7 m) at time $t = 2$ s and
(13 m, 14 m) at time $t = 5$ s.
Average velocity vector $(\vec{v}_{av})$ from $t = 0$ to $t = 5$ s is
(a) $\frac{1}{5}(13\hat{i}+14\hat{j})$ (b) $\frac{7}{3}(\hat{i}+\hat{j})$
(c) $2(\hat{i}+\hat{j})$ (d) $\frac{11}{5}(\hat{i}+\hat{j})$

4. A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction $= \mu$). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is
(Assume $m_1 = m_2 = m_3 = m$)



(a) $\frac{g(1-g\mu)}{9}$ (b) $\frac{2g\mu}{3}$
(c) $\frac{g(1-2\mu)}{3}$ (d) $\frac{g(1-2\mu)}{2}$

5. The force F acting on a particle of mass m is indicated by the force-time graph shown below. The change in momentum of the particle over the time interval from zero to 8 s is



(a) 24 N s (b) 20 N s
(c) 12 N s (d) 6 N s

6. A balloon with mass m is descending down with an acceleration a (where $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a?
(a) $\frac{2ma}{g+a}$ (b) $\frac{2ma}{g-a}$
(c) $\frac{ma}{g+a}$ (d) $\frac{ma}{g-a}$

7. A body of mass $(4m)$ is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces, each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is
(a) mv^2 (b) $\frac{3}{2}mv^2$
(c) $2mv^2$ (b) $4mv^2$

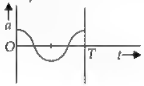
8. The oscillation of a body on a smooth horizontal surface is represented by the equation,

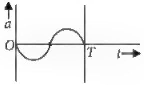
$X = A\cos(\omega t)$

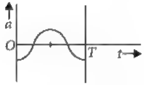
where X = displacement at time t

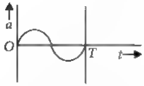
ω = frequency of oscillation

Which one of the following graphs shows correctly the variation a with t?

(a) 

(b) 

(c) 

(d) 

Here a = acceleration at time t

T = time period

9. A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions s^{-2} is

(a) 25 N (b) 50 N (c) 78.5 N (d) 157 N

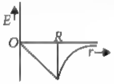
10. The ratio of the accelerations for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is

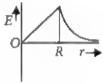
(a) 5 : 7 (b) 2 : 3 (c) 2 : 5 (d) 7 : 5

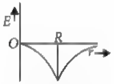
11. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole?

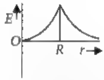
(a) 10^{-9} m (b) 10^{-6} m
(c) 10^{-2} m (d) 100 m

12. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by

(a) 

(b) 

(c) 

(d) 

13. Copper of fixed volume V is drawn into wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. Which of the following graphs is a straight line?

(a) Δl versus $1/l$ (b) Δl versus l^2
(c) Δl versus $1/l^2$ (d) Δl versus l

14. A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then

(a) energy $= 4VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released.

(b) energy $= 3VT\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed.

(c) energy $= 3VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released.

(d) energy is neither released nor absorbed.

15. Steam at 100°C is passed into 20 g of water at 10°C. When water acquires a temperature of 80°C, the mass of water present will be
[Take specific heat of water = 1 cal g^{-1} °C^{-1} and latent heat of steam = 540 cal g^{-1}]

(a) 24 g (b) 31.5 g (c) 42.5 g (d) 22.5 g

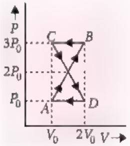
16. Certain quantity of water cools from 70°C to 60°C in the first 5 minutes and to 54°C in the next 5 minutes. The temperature of the surroundings is

(a) 45°C (b) 20°C (c) 42°C (d) 10°C

17. A monoatomic gas at a pressure P, having a volume V expands isothermally to a volume

$2V$ and then adiabatically to a volume $16V$. The final pressure of the gas is (Take $\gamma = 5/3$)
(a) $64P$ (b) $32P$ (c) $P/64$ (d) $16P$

18. A thermodynamic system undergoes cyclic process $ABCDA$ as shown in figure. The work done by the system in the cycle is



(a) P_0V_0 (b) $2P_0V_0$ (c) $\frac{P_0V_0}{2}$ (d) zero

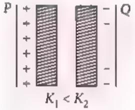
19. The mean free path of molecules of a gas, (radius r) is inversely proportional to
(a) r^3 (b) r^2 (c) r (d) $\sqrt{r}$

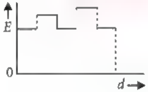
20. If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by
(a) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$
(b) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$
(c) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$
(d) $n = n_1 + n_2 + n_3$

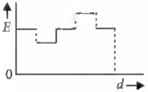
21. The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are (Velocity of sound = 340 m s^{-1})
(a) 4 (b) 5 (c) 7 (d) 6

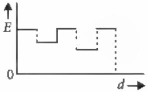
22. A speeding motorcyclist sees traffic jam ahead him. He slows down to 36 km hour^{-1}. He finds that traffic has eased and a car moving ahead of him at 18 km hour^{-1} is honking at a frequency of 1392 Hz. If the speed of sound is 343 m s^{-1}, the frequency of the honk as heard by him will be
(a) 1332 Hz (b) 1372 Hz
(c) 1412 Hz (d) 1454 Hz

23. Two thin dielectric slabs of dielectric constants K_1 and K_2 ($K_1 < K_2$) are inserted between plates of a parallel plate capacitor, as shown in the figure. The variation of electric field E between the plates with distance d as measured from plate P is correctly shown by



(a)


(b)


(c)


(d)

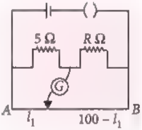

24. A conducting sphere of radius R is given a charge Q. The electric potential and the electric field at the centre of the sphere respectively are
(a) zero and $\frac{Q}{4\pi\varepsilon_0 R^2}$
(b) $\frac{Q}{4\pi\varepsilon_0 R}$ and zero
(c) $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{Q}{4\pi\varepsilon_0 R^2}$
(d) both are zero

25. In a region, the potential is represented by $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where V is in volts and x, y, z are in metres. The electric force experienced by a charge of 2 coulomb situated at point (1, 1, 1) is

(a) $6\sqrt{5}$ N (b) 30 N
(c) 24 N (d) $4\sqrt{35}$ N

26. Two cities are 150 km apart. Electric power is sent from one city to another city through copper wires. The fall of potential per km is 8 volt and the average resistance per km is 0.5 Ω. The power loss in the wire is
(a) 19.2 W (b) 19.2 kW
(c) 19.2 J (d) 12.2 kW

27. The resistances in the two arms of the meter bridge are 5 Ω and R Ω respectively. When the resistance R is shunted with an equal resistance, the new balance point is at $1.6l_1$. The resistance R is



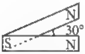
(a) 10 Ω (b) 15 Ω (c) 20 Ω (d) 25 Ω

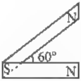
28. A potentiometer circuit has been set up for finding the internal resistance of a given cell. The main battery, used across the potentiometer wire, has an emf of 2.0 V and a negligible internal resistance. The potentiometer wire itself is 4 m long. When the resistance R, connected across the given cell, has values of
(i) infinity (ii) 9.5 Ω
the balancing lengths on the potentiometer wire are found to be 3 m and 2.85 m, respectively. The value of internal resistance of the cell is
(a) 0.25 Ω (b) 0.95 Ω (c) 0.5 Ω (d) 0.75 Ω

29. Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment $\vec{m}$. Which configuration has highest net magnetic dipole moment?

(1) 

(2) 

(3) 

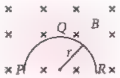
(4) 

(a) (1) (b) (2) (c) (3) (d) (4)

30. In an ammeter 0.2% of main current passes through the galvanometer. If resistance of galvanometer is G, the resistance of ammeter will be
(a) $\frac{1}{499}G$ (b) $\frac{499}{500}G$
(c) $\frac{1}{500}G$ (d) $\frac{500}{499}G$

31. Two identical long conducting wires AOB and COD are placed at right angle to each other, with one above other such that O is their common point for the two. The wires carry I_1 and I_2 currents, respectively. Point P is lying at distance d from O along a direction perpendicular to the plane containing the wires. The magnetic field at the point P will be
(a) $\frac{\mu_0}{2\pi d}\left(\frac{I_1}{I_2}\right)$ (b) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$
(c) $\frac{\mu_0}{2\pi d}(I_1^2 - I_2^2)$ (d) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$

32. A thin semicircular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.



The potential difference developed across the ring when its speed is v, is
(a) zero
(b) $\frac{Bv\pi r^2}{2}$ and P is at higher potential
(c) πrBv and R is at higher potential
(d) $2rBv$ and R is at higher potential

33. A transformer having efficiency of 90% is working on 200 V and 3 kW power supply. If the current in the secondary coil is 6 A, the voltage across the secondary coil and the current in the primary coil respectively are
(a) 300 V, 15 A (b) 450 V, 15 A
(c) 450 V, 13.5 A (d) 600 V, 15 A

34. Light with an energy flux of 25×10^4 W m^{-2} falls on a perfectly reflecting surface at normal

incidence. If the surface area is 15 cm^2, the average force exerted on the surface is

(a) 1.25×10^{-6} N (b) 2.50×10^{-6} N
(c) 1.20×10^{-6} N (d) 3.0×10^{-6} N

35. A beam of light of λ = 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between first dark fringes on either side of the central bright fringe is

(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

36. In the Young's double slit experiment, the intensity of light at a point on the screen where the path difference λ is K, (λ being the wavelength of light used). The intensity at a point where the path difference is $\lambda/4$ will be

(a) K (b) $K/4$ (c) $K/2$ (d) zero

37. If the focal length of objective lens is increased then magnifying power of

(a) microscope will increase but that of telescope decrease.
(b) microscope and telescope both will increase.
(c) microscope and telescope both will decrease.
(d) microscope will decrease but that of telescope will increase.

38. The angle of a prism is A. One of its refracting surfaces is silvered. Light rays falling at an angle of incidence $2A$ on the first surface returns back through the same path after suffering reflection at the silvered surface. The refractive index μ, of the prism is

(a) $2\sin A$ (b) $2\cos A$
(c) $\frac{1}{2}\cos A$ (d) $\tan A$

39. When the energy of the incident radiation is increased by 20%, the kinetic energy of the photoelectrons emitted from a metal surface increased from 0.5 eV to 0.8 eV. The work function of the metal is

(a) 0.65 eV (b) 1.0 eV
(c) 1.3 eV (d) 1.5 eV

40. If the kinetic energy of the particle is increased to 16 times its previous value, the percentage change in the de Broglie wavelength of the particle is

(a) 25 (b) 75 (c) 60 (d) 50

41. Hydrogen atom in ground state is excited by a monochromatic radiation of λ = 975 Å. Number of spectral lines in the resulting spectrum emitted will be

(a) 3 (b) 2 (c) 6 (d) 10

42. The binding energy per nucleon of ^7_3Li and ^4_2He nuclei are 5.60 MeV and 7.06 MeV respectively. In the nuclear reaction

$$^7_3Li + ^1_1H \rightarrow ^4_2He + ^4_2He + Q$$

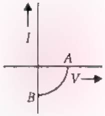
the value of energy Q released is

(a) 19.6 MeV (b) –2.4 MeV
(c) 8.4 MeV (d) 17.3 MeV

43. A radioisotope X with a half life 1.4×10^9 years decays to Y which is stable. A sample of the rock from a cave was found to contain X and Y in the ratio 1 : 7. The age of the rock is

(a) 1.96×10^9 years (b) 3.92×10^9 years
(c) 4.20×10^9 years (d) 8.40×10^9 years

44. The given graph represents V-I characteristic for a semiconductor device.



Which of the following statement is correct?

(a) It is V-I characteristic for solar cell where, point A represents open circuit voltage and point B short circuit current.
(b) It is for a solar cell and points A and B represent open circuit voltage and current, respectively.
(c) It is for a photodiode and points A and B represent open circuit voltage and current, respectively.
(d) It is for a LED and points A and B represent open circuit voltage and short circuit current, respectively.

45. The barrier potential of a p-n junction depends on
(1) type of semiconductor material
(2) amount of doping
(3) temperature
Which one of the following is correct?
(a) (1) and (2) only (b) (2) only
(c) (2) and (3) only (d) (1), (2) and (3)

SOLUTIONS

1. (d) : Let mass $m \propto F^a V^b T^c$

or $m = kF^a V^b T^c$ ...(i)

where k is a dimensionless constant and a, b and c are the exponents.

Writing dimensions on both sides, we get

$[ML^0T^0] = [MLT^{-2}]^a [LT^{-1}]^b [T]^c$

$[ML^0T^0] = [M^aL^{a+b}T^{-2a-b+c}]$

Applying the principle of homogeneity of dimensions, we get

$a = 1$ ... (ii)

$a + b = 0$ ... (iii)

$-2a - b + c = 0$ ... (iv)

Solving eqns. (ii), (iii) and (iv), we get

$a = 1, b = -1, c = 1$

From eqn. (i), $[m] = [FV^{-1}T]$

2. (a) : The equation of trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

where θ is the angle of projection and u is the velocity with which projectile is projected.

For equal trajectories for same angles of projection,

$$\frac{g}{u^2} = \text{constant}$$

As per question, $\frac{9.8}{5^2} = \frac{g'}{3^2}$

where g' is acceleration due to gravity on the planet.

$$g' = \frac{9.8\times 9}{25} = 3.5 \text{ m s}^{-2}$$

3. (d) : At time $t = 0$, the position vector of the particle is

$\vec{r}_1 = 2\hat{i} + 3\hat{j}$

At time $t = 5$ s, the position vector of the particle is

$\vec{r}_2 = 13\hat{i} + 14\hat{j}$

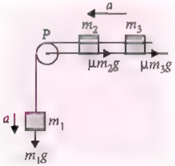
Displacement from $\vec{r}_1$ to $\vec{r}_2$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (13\hat{i} + 14\hat{j}) - (2\hat{i} + 3\hat{j})$$
$$= 11\hat{i} + 11\hat{j}$$

$\therefore$ Average velocity,

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{11\hat{i} + 11\hat{j}}{5-0} = \frac{11}{5}(\hat{i} + \hat{j})$$

4. (c):



Force of friction on mass $m_2 = \mu m_2 g$

Force of friction on mass $m_3 = \mu m_3 g$

Let a be common acceleration of the system.

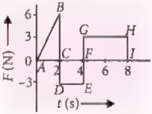
$$\therefore \quad a = \frac{m_1 g - \mu m_2 g - \mu m_3 g}{m_1 + m_2 + m_3}$$

Here, $m_1 = m_2 = m_3 = m$

$$\therefore \quad a = \frac{mg - \mu mg - \mu mg}{m + m + m} = \frac{mg - 2\mu mg}{3m}$$
$$= \frac{g(1-2\mu)}{3}$$

Hence, the downward acceleration of mass m_1 is $\frac{g(1-2\mu)}{3}$.

5. (c):



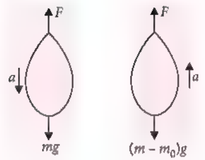
Change in momentum = Area under F-t graph in that interval

= Area of ΔABC – Area of rectangle $CDEF$ + Area of rectangle $FGHI$

$$= \frac{1}{2}\times 2\times 6 - 3\times 2 + 4\times 3 = 12 \text{ N s}$$

6. (a) : Let F be the upthrust of the air. As the balloon is descending down with an acceleration a,

$\therefore \quad mg - F = ma$ ... (i)



Let mass m_0 be removed from the balloon so that it starts moving up with an acceleration a. Then,

$$F - (m - m_0)g = (m - m_0)a$$

$$F - mg + m_0g = ma - m_0a \quad \text{... (ii)}$$

Adding eqn. (i) and eqn. (ii), we get

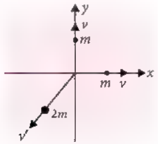
$$m_0g = 2ma - m_0a$$

$$m_0g + m_0a = 2ma$$

$$m_0(g + a) = 2ma$$

$$m_0 = \frac{2ma}{a+g}$$

7. **(b) :**



Let $\vec{v}'$ be velocity of third piece of mass $2m$.

Initial momentum, $\vec{p}_i = 0$ (As the body is at rest)

Final momentum, $\vec{p}_f = mv\hat{i} + mv\hat{j} + 2m\vec{v}'$

According to law of conservation of momentum

$$\vec{p}_i = \vec{p}_f$$

$$0 = mv\hat{i} + mv\hat{j} + 2m\vec{v}'$$

$$\vec{v}' = -\frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$$

The magnitude of $\vec{v}'$ is

$$v' = \sqrt{\left(-\frac{v}{2}\right)^2 + \left(-\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$$

Total kinetic energy generated due to explosion

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2$$

$$= mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

8. **(c):** Here, $X = A\cos\omega t$

$\therefore$ Velocity, $v = \frac{dX}{dt} = \frac{d}{dt}(A\cos\omega t)$

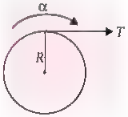
$$= -A\omega\sin\omega t$$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega\sin\omega t)$

$$= -A\omega^2\cos\omega t$$

Hence the variation of a with t is correctly shown by graph (c).

9. **(d) :**



Here, mass of the cylinder, $M = 50$ kg

Radius of the cylinder, $R = 0.5$ m

Angular acceleration, $\alpha = 2$ rev s^{-2}

$= 2 \times 2\pi$ rad s^{-2} = 4π rad s^{-2}

Torque, $\tau = TR$

Moment of inertia of the solid cylinder about its axis, $I = \frac{1}{2}MR^2$

$\therefore$ Angular acceleration of the cylinder

$$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$$

$$T = \frac{MR\alpha}{2} = \frac{50 \times 0.5 \times 4\pi}{2} = 157 \text{ N}$$

10. **(a) :** Acceleration of the solid sphere slipping down the incline without rolling is

$$a_{slipping} = g\sin\theta \quad \text{... (i)}$$

Acceleration of the solid sphere rolling down the incline without slipping is

$$a_{rolling} = \frac{g\sin\theta}{1+\frac{k^2}{R^2}} = \frac{g\sin\theta}{1+\frac{2}{5}}$$

$$\left(\because \text{For solid sphere, } \frac{k^2}{R^2} = \frac{2}{5}\right)$$

$$= \frac{5}{7}g\sin\theta \quad \text{... (ii)}$$

Divide eqn. (ii) by eqn. (i), we get

$$\frac{a_{\text{rolling}}}{a_{\text{slipping}}} = \frac{5}{7}$$

11. **(c)**: Light cannot escape from a black hole, $v_{esc} = c$

$$\sqrt{\frac{2GM}{R}} = c \quad \text{or} \quad R = \frac{2GM}{c^2}$$

$$R = \frac{2 \times 6.67 \times 10^{-11}\ \text{N m}^2\text{kg}^{-2} \times 5.98 \times 10^{24}\,\text{kg}}{(3 \times 10^8\ \text{m s}^{-1})^2}$$

$$= 8.86 \times 10^{-3}\ \text{m} \approx 10^{-2}\ \text{m}$$

12. **(a)** : For a point inside the earth *i.e.* $r < R$

$$E = -\frac{GM}{R^3} r$$

where M and R be mass and radius of the earth respectively.

At the centre, $r = 0$

$\therefore \quad E = 0$

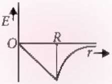
For a point outside the earth *i.e.* $r > R$,

$$E = -\frac{GM}{r^2}$$

On the surface of the earth *i.e.* $r = R$,

$$E = -\frac{GM}{R^2}$$

The variation of E with distance r from the centre is as shown in the figure.



13. **(b)** : As $V = Al$ ... (i)

where A is the area of cross-section of the wire.

$$\text{Young's modulus, } Y = \frac{(F/A)}{(\Delta l / l)} = \frac{Fl}{A\Delta l}$$

$$\Delta l = \frac{Fl}{YA} = \frac{Fl^2}{YV} \qquad \text{(Using (i))}$$

$\Delta l \propto l^2$

Hence, the graph between Δl and l^2 is a straight line.

14. **(c)**: Let n droplets each of radius r coalesce to form a big drop of radius R.

$\therefore$ Volume of n droplets = Volume of big drop

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$n = \frac{R^3}{r^3} \qquad \text{... (i)}$$

Volume of big drop, $V = \frac{4}{3}\pi R^3$ ... (ii)

Initial surface area of n droplets,

$$A_i = n \times 4\pi r^2 = \frac{R^3}{r^3} \times 4\pi r^2 \qquad \text{(Using (i))}$$

$$= 4\pi \frac{R^3}{r} = \left(\frac{4}{3}\pi R^3\right)\frac{3}{r}$$

$$= \frac{3V}{r} \qquad \text{(Using (ii))}$$

Final surface area of big drop

$$A_f = 4\pi R^2 = \left(\frac{4}{3}\pi R^3\right)\frac{3}{R} = \frac{3V}{R} \qquad \text{(Using (ii))}$$

Decrease in surface area

$$\Delta A = A_i - A_f = \frac{3V}{r} - \frac{3V}{R} = 3V\left(\frac{1}{r} - \frac{1}{R}\right)$$

$\therefore$ Energy released = Surface tension × Decrease in surface area

$= T \times \Delta A$

$= 3VT\left(\frac{1}{r} - \frac{1}{R}\right)$

15. **(d)** : Here,

Specific heat of water, $s_w = 1$ cal g^{-1} °C^{-1}

Latent heat of steam, $L_s = 540$ cal g^{-1}

Heat lost by m g of steam at 100°C to change into water at 80°C is

$$Q_1 = mL_s + ms_w\Delta T_w$$

$$= m \times 540 + m \times 1 \times (100 - 80)$$

$$= 540m + 20m = 560m$$

Heat gained by 20 g of water to change its temperature from 10°C to 80°C is

$$Q_2 = m_w s_w \Delta T_w = 20 \times 1 \times (80 - 10) = 1400$$

According to principle of calorimetry

$$Q_1 = Q_2$$

$\therefore \quad 560m = 1400$ or $m = 2.5$ g

Total mass of water present

$= (20 + m)$ g $= (20 + 2.5)$ g $= 22.5$ g

16. **(a)** : Let T_s be the temperature of the surroundings.

According to Newton's law of cooling

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$$

For first 5 minutes,

$T_1 = 70°C$, $T_2 = 60°C$, $t = 5$ minutes

$$\therefore \quad \frac{70-60}{5} = K\left(\frac{70+60}{2} - T_s\right)$$

$$\frac{10}{5} = K(65 - T_s) \qquad \text{... (i)}$$

For next 5 minutes,

$T_1 = 60°C$, $T_2 = 54°C$, $t = 5$ minutes

$$\therefore \quad \frac{60-54}{5} = K\left(\frac{60+54}{2} - T_s\right)$$

$$\frac{6}{5} = K(57 - T_s) \qquad \text{... (ii)}$$

Divide eqn. (i) by eqn. (ii), we get

$$\frac{5}{3} = \frac{65 - T_s}{57 - T_s}$$

$285 - 5T_s = 195 - 3T_s$

$2T_s - 90$ or $T_s = 45°C$

17. (c): First, isothermal expansion

$PV - P'(2V)$

(For isothermal process, PV = constant)

$$P' = \frac{P}{2}$$

Then, adiabatic expansion

$P'(2V)^\gamma = P_f(16V)^\gamma$

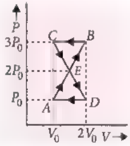
(For adiabatic process, PV^γ = constant)

$$\frac{P}{2}(2V)^{5/3} = P_f\,(16V)^{5/3}$$

$$P_f = \frac{P}{2}\left(\frac{2V}{16V}\right)^{5/3} = \frac{P}{2}\left(\frac{1}{8}\right)^{5/3} = \frac{P}{2}\left(\frac{1}{2^3}\right)^{5/3}$$

$$= \frac{P}{2}\left(\frac{1}{2^5}\right) = \frac{P}{64}$$

18. (d) : In a cyclic process work done is equal to the area under the cycle and is positive if the cycle is clockwise and negative if anticlockwise.



As is clear from figure,

$$W_{AEDA} = +\text{area of } \Delta AED = +\frac{1}{2}\,P_0V_0$$

$$W_{BCEB} = -\text{Area of } \Delta BCE = -\frac{1}{2}\,P_0V_0$$

The net work done by the system is

$$W_{\text{net}} = W_{AEDA} + W_{BCEB}$$

$$= +\frac{1}{2}P_0V_0 - \frac{1}{2}P_0V_0 = \text{zero}$$

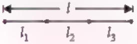
19. (b) : Mean free path, $\lambda = \dfrac{1}{\sqrt{2}n\pi d^2}$

where n is the number density and d is the diameter of the molecule.

As $d = 2r$

$$\therefore \quad \lambda = \frac{1}{4\sqrt{2}n\pi r^2} \quad \text{or} \quad \lambda \propto \frac{1}{r^2}$$

20. (a) :



$$n_1 = \frac{1}{2l_1}\sqrt{\frac{T}{\mu}} \qquad \text{...(i)}$$

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T}{\mu}} \qquad \text{...(ii)}$$

$$n_3 = \frac{1}{2l_3}\sqrt{\frac{T}{\mu}} \qquad \text{...(iii)}$$

$$n = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \qquad \text{...(iv)}$$

From eqn. (iv), we get

$$\frac{1}{n} = \frac{2l}{\sqrt{\frac{T}{\mu}}}$$

As $l = l_1 + l_2 + l_3$

$$\therefore \quad \frac{1}{n} = \frac{2(l_1 + l_2 + l_3)}{\sqrt{\frac{T}{\mu}}} = \frac{2l_1}{\sqrt{\frac{T}{\mu}}} + \frac{2l_2}{\sqrt{\frac{T}{\mu}}} + \frac{2l_3}{\sqrt{\frac{T}{\mu}}}$$

$$= \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} \quad \text{[Using (i), (ii) and (iii)]}$$

21. (d)	22. (c)	23. (c)	24. (b)	25. (d)
26. (b)	27. (b)	28. (c)	29. (c)	30. (c)
31. (d)	32. (d)	33. (b)	34. (b)	35. (d)
36. (c)	37. (d)	38. (b)	39. (b)	40. (b)
41. (c)	42. (d)	43. (c)	44. (a)	45. (d)

*For detailed solutions please refer to **'MTG AIPMT Explorer'**.*

NCERT Xtract

New

Questions for Medical/ Engineering Entrance Exams

Ray Optics and Wave Optics

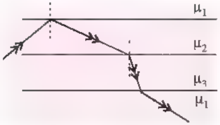
1. A person wants a real image of his own, 3 times enlarged. Where should he stand before a concave mirror of radius of curvature 30 cm?
   (a) 10 cm (b) 30 cm
   (c) 90 cm (d) 20 cm

2. A metal plate is lying at the bottom of a tank full of a transparent liquid. Height of tank is 100 cm but the plate appears to be at 45 cm above bottom. The refractive index of liquid is
   (a) 1.00 (b) 1.53
   (c) 1.81 (d) 1.32

3. A ray of light when falls on a liquid A – air interface at angle 45°, it just gets total reflection. When a ray of light falls on liquid B – air interface at angle 30°, it does not emerge out. We can infer that
   (a) $\mu_A > \mu_B$ (b) $\mu_A < \mu_B$
   (c) $\mu_A = \mu_B$ (d) can't be inferred

4. A ray of light falls on a glass slab making an angle of 45° with normal. The emergent ray makes the same angle with normal. It shows that
   (a) there is no refraction taking place
   (b) incident ray is perpendicular to emergent ray
   (c) emergent ray is parallel to the incident ray
   (d) the given case is not possible.

5. Figure shows two glass slabs placed one above the other in air. Refractive indices of glass slabs are μ_2 and μ_3 and that of air is μ_1. A ray of light enters and follows a path as shown in figure. It can be concluded that



   (a) $\mu_1 = \mu_2 = \mu_3$ (b) $\mu_1 < \mu_2 < \mu_3$
   (c) $\mu_1 > \mu_2 = \mu_3$ (d) $\mu_1 > \mu_2 > \mu_3$

6. Two thin convex lenses of focal length 20 cm and 25 cm are placed at a finite distance apart. The power of the combination is 8 D. Distance between the lenses is
   (a) 5 cm (b) 10 cm
   (c) 8 cm (d) 4 cm

7. When a ray of light falls on a prism of angle 48°, it suffers minimum deviation. Refractive index of the material of prism is 1.6. The angle of incidence is nearly
   (a) 16° (b) 48° (c) 30° (d) 41°

8. When white light passes through a prism, it splits into its constituent colours . Which colour suffers the maximum deviation?
   (a) Red (b) Violet
   (c) Green (d) Blue

9. A ray of light incident on a prism of angle 60°, does not emerge out. Refractive index of material of prism may be

(a) 1.50 (b) 2.30
(c) 1.80 (d) 1.20

10. When light travels from a rarer to a denser medium, it loses some speed. Energy carried by the light wave is

(a) greater in rarer medium
(b) greater in denser medium
(c) equal in both mediums
(d) can not be said

11. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 8.1 mm. A second light produces an interference pattern in which the fringes are separated by 7.2 mm. The wavelength of the second light is

(a) 420 nm (b) 560 nm
(c) 810 nm (d) 980 nm

12. In Young's double slit experiment, light of wavelength 6000 Å is used to get an interference pattern on a screen. The fringe width changes by 1.5 mm, when the screen is brought towards the double slit by 50 cm. The distance between the two slits is

(a) 0.2 mm (b) 0.4 mm
(c) 0.6 mm (d) 0.8 mm

13. If ε_0 and μ_0 are the permittivity and permeability of free space and ε and μ are corresponding quantities for a medium, then refractive index of the medium is

(a) $\sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}$ (b) $\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$
(c) 1
(d) insufficient information

14. Out of the following, which can be used as coherent sources required to produce interference pattern of light ?

(a) Two identical and independent sodium lamps
(b) One yellow light and one green light source
(c) Two light sources with unequal amplitude
(d) None of these

15. The polarising angle for a medium is 60°. The critical angle for the medium is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(c) $\cos^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

16. A beam of light travelling in water falls on a glass plate immersed in water. When the incident angle is 51°, the reflected beam of light is found to be completely plane polarised. The refractive index of glass is
(Given refractive index of water = 4/3)

(a) 1.091 (b) 2.312 (c) 1.647 (d) 5.734

17. Which of the following waves cannot be polarised ?

(a) Radiowaves (b) Infrared waves
(c) Ultrasonic waves (d) Microwaves

18. If a concave mirror is held in water, then its focal length

(a) increases
(b) decreases
(c) remains unchanged
(d) may increase or decrease depending upon the level of water

19. If a convex lens is immersed in water, then its focal length

(a) increases
(b) decreases
(c) remains unchanged
(d) may increase or decrease depending upon the level of water

20. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen, then

(a) half the image will disappear
(b) no image will be formed
(c) intensity of image will decrease
(d) intensity of image will increase

21. In the Young's double slit experiment using a monochromatic light of wavelength λ, the

path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$

(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

22. A girl wearing coloured contact lens, blue for the left eye and green for the right eye, is standing in front of a plane mirror. In the image, she observes
(a) the left eye as green and right eye as blue
(b) the left eye as blue and right eye as green
(c) both the eyes as green
(d) both the eyes as blue.

HIGHER ORDER THINKING SKILLS QUESTIONS (HOTS)

23. Diffraction pattern from a single slit of width 0.25 mm is observed with light of wavelength 5890 Å. Angular separation between first order minimum and third order maximum, falling on the same side, is
(a) 5.89×10^{-3} rad (b) 5.89×10^{-7} rad
(c) 5.89×10^{-10} rad (d) 5.89×10^{-4} rad

24. Wavelength of light coming from two stars is same as the de-Broglie wavelength associated with an electron accelerated through a potential difference of 100 V. The aperture of objective of a telescope that can be used to just resolve the stars separated by 9×10^{-9} rad, is
(a) 16.7 mm (b) 98.3 mm
(c) 30.8 mm (d) 42.5 mm

25. A beam of unpolarised light is incident on two polaroids crossed to each other. When one of the polaroid is rotated through an angle, then 25% of the incident unpolarised light is transmitted by the polaroids. Then the angle through which polaroid is rotated, is
(a) 25° (b) 45°
(c) 0° (d) 90°

26. A point source of light is placed at a depth of h below the surface of water of refractive index μ. A floating opaque disc is placed on the surface of water so that light from the source is not visible from the surface. The minimum diameter of the disc is

(a) $\frac{2h}{(\mu^2-1)^{1/2}}$ (b) $2h(\mu^2-1)^{1/2}$

(c) $\frac{h}{2(\mu^2-1)^{1/2}}$ (d) $h(\mu^2-1)^{1/2}$

27. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure. A polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.

Which of the following statement is true?
(a) For a particular orientation there shall be darkness as observed through the polaroid.
(b) The intensity of light as seen through the polaroid shall be independent of the rotation.
(c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid.
(d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid.

28. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. The magnifying power of the telescope for viewing distant objects when the final image is formed at the least distance of distinct vision 25 cm is
(a) 33.6 (b) 66.12 (c) 22.6 (d) 11.6

29. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? (Take wavelength of light = 500 nm).
(a) 5 m (b) 1 m (c) 6 m (d) 3 m

30. The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ rad and typical photoprinter prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance z should a printed page be held so that one does not see the individual dots?
(a) 14.5 cm (b) 20.5 cm
(c) 29.5 cm (d) 28 cm

SOLUTIONS

1. (a) : Let the object distance be u
$R = -30$ cm or $f = -15$ cm
$$m = \frac{h_I}{h_O} = \frac{-v}{u} = 3 \quad \Rightarrow v = -3u$$
Using mirror formula,
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-15} = \frac{1}{-3u} + \frac{1}{u}$$
$$\text{or } \frac{2}{3u} = \frac{-1}{15} \Rightarrow u = \frac{-2\times 15}{3} = -10 \text{ cm}$$

2. (c): Real depth of plate, $H = 100$ cm
Apparent depth of plate, $h = 100 - 45 = 55$ cm
$$\therefore \text{ Refractive index of fluid} = \frac{H}{h} = \frac{100}{55} = 1.81$$

3. (b) : For liquid A – air interface,
$$\mu_A = \frac{1}{\sin i_C} = \frac{1}{\sin 45°} = \sqrt{2}$$
For liquid B – air interface, the ray does not emerge out for $\theta = 30°$ *i.e.*, $i_c \le 30°$
$$\sin i_c \le \sin 30° = \frac{1}{2} \text{ or } \frac{1}{\sin i_c} \ge \frac{1}{2} \quad \Rightarrow \mu \ge 2°$$

4. (c)

5. (b) : As the incident ray gets internally reflected at $\mu_1 - \mu_2$ interface,
$\therefore \mu_2 > \mu_1$
Now, at $\mu_2 - \mu_3$ interface, the refracted ray bends towards the normal,
$\therefore \mu_3 > \mu_2$
Now, at $\mu_3 - \mu_1$ interface, refracted ray move away from normal,
$\therefore \mu_3 > \mu_1$
So, $\mu_3 > \mu_2 > \mu_1$

6. (a) : Here, $f_1 = 20$ cm, $f_2 = 25$ cm, $P = 8$ D
$\Rightarrow$ Focal length of combination,
$$= \frac{1}{8} \text{ m} = \frac{100}{8} \text{ cm}$$
Now, using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$,
(where d is the distance between lenses), we get
$$\frac{8}{100} = \frac{1}{20} + \frac{1}{25} - \frac{d}{20\times 25}$$
$$\Rightarrow \frac{d}{500} = \frac{1}{20} + \frac{1}{25} - \frac{8}{100} = \frac{5+4-8}{100} = \frac{1}{100}$$
$$\text{or } d = \frac{500}{100} = 5 \text{ cm}$$

7. (d) : Given, $A = 48°$, $\mu = 1.6$
For minimum deviation, $r = \frac{A}{2}$
According to Snell's law, $\mu = \frac{\sin i}{\sin r}$
$$\text{or } \sin i = \mu \sin r = \mu \sin\frac{A}{2}$$
$$= 1.6 \times \sin\frac{48°}{2} = 1.6 \times \sin 24°$$
$$= 1.6 \times 0.407 = 0.651$$
$\therefore i = \sin^{-1}(0.651) = 40.6° \approx 41°$

8. (b) : As $\mu_{violet} > \mu_{blue} > \mu_{green} > \mu_{red}$
Therefore, $\delta_{violet} > \delta_{blue} > \delta_{green} > \delta_{red}$
[$\because \delta = (\mu - 1)A$]

9. (b) : For no emergence, $\mu > \frac{1}{\sin A/2}$
Here, $A = 60°$
$$\therefore \mu > \frac{1}{\sin 30°} = 2$$
Hence, 2.30 may be the refractive index of material of prism.

10. (c): When light travels from one medium to another, speed and wavelength changes but frequency remains the same. Therefore, energy carried by the light wave remains same.

11. **(b)** : Here $\lambda_1 = 630$ nm, $\beta_1 = 8.1$ mm, $\beta_2 = 7.2$ mm

Fringe width, $\beta = \frac{D\lambda}{d}$

For constant D and d, $\beta \propto \lambda$

$\therefore \quad \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$

$\therefore \quad \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1 = \frac{7.2 \text{ mm}}{8.1 \text{ mm}} \times 630 \text{ nm} = 560 \text{ nm}.$

12. **(a)** : Fringe width, $\beta = \frac{D\lambda}{d}$

As λ and d are constant, so the change in fringe width is given by

$\Delta\beta = \frac{\lambda \Delta D}{d}$

$\therefore$ Distance between the two slits,

$d = \frac{\lambda \Delta D}{\Delta\beta} = \frac{6000 \times 10^{-10} \times 50 \times 10^{-2}}{1.5 \times 10^{-3}}$

$= 0.2 \times 10^{-3}$ m $= 0.2$ mm.

13. **(b)** : Velocity of light in vacuum, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Velocity of light in medium, $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$\therefore$ Refractive index of the medium,

$\mu = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}}$

14. **(d)**

15. **(d)** : Here, $i_p = 60°$

Refractive index of the medium,

$\mu = \tan i_p = \tan 60° = \sqrt{3}$

Now, critical angle of the medium, i_c

$= \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

16. **(c)**: Refractive index of glass w.r.t water,

${}^w\mu_g = \tan i_p = \tan 51° = 1.235$

$\therefore \quad {}^a\mu_g = {}^w\mu_g \times {}^a\mu_w = 1.235 \times \frac{4}{3} = 1.647$

17. **(c)**: Ultrasonic waves are longitudinal in nature, so they cannot be polarised.

18. **(c)**: The focal length of a concave mirror does not depend on the nature of the medium.

19. **(a)** : Focal length of a convex lens, $f \propto \frac{1}{\mu - 1}$

As ${}^w\mu_g < {}^a\mu_g$, so focal length of the convex lens increases as it is immersed in water.

20. **(c)**: Image formed is complete but has decreased intensity.

21. **(b)** : As $I = I_{max} \cos^2\left(\frac{\phi}{2}\right)$

Here, $I = \frac{I_{max}}{2}$

$\therefore \quad \frac{I_{max}}{2} = I_{max} \cos^2\left(\frac{\phi}{2}\right) \Rightarrow \frac{1}{2} = \cos^2\frac{\phi}{2}$

$\Rightarrow \cos\phi = 0 \Rightarrow \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

or $\phi = (2n+1)\frac{\pi}{2}$ where $n = 0, 1, 2, \ldots$

Since phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

$\therefore$ Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference

$= \frac{\lambda}{2\pi} \times (2n+1)\frac{\pi}{2} = (2n+1)\frac{\lambda}{4}$

22. **(a)** : A plane mirror will produce a virtual but laterally inverted image of any object placed at any distance from the mirror. The lateral inversion makes the left side appear as the right side and the right side as left. Thus the girl observes the left eye as green in the image and the right eye as blue.

23. **(a)** : In single slit diffraction pattern, positions of secondary maxima and minima are given by,

$\theta_{max} = \pm(2n+1)\frac{\lambda}{2d}$ and $\theta_{min} = \pm n\frac{\lambda}{d}$, respectively.

$\therefore \quad \theta_{1\,min} = \frac{\lambda}{d}$ and $\theta_{3\,max} = \frac{7\lambda}{2d}$

Angular separation $= \theta_{3\,max} - \theta_{1\,min} = \frac{7\lambda}{2d} - \frac{\lambda}{d}$

$= \frac{5\lambda}{2d} = \frac{5}{2} \times \frac{5890 \times 10^{-10}}{0.25 \times 10^{-3}} = 5.89 \times 10^{-3}$ rad.

24. **(a)** : de-Broglie wavelength associated with an electron accelerated through a potential difference is

Contd. on page no. 89

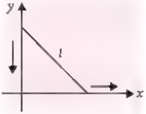
PRACTICE QUESTIONS

TARGET PMTs

Useful for All National and State Level PMTs

1. The magnitude of acceleration of a particle as seen by observer A is a m s^{-2} and that observed by B is b m s^{-2}. If magnitude of acceleration of A with respect to B is x m s^{-2}, then the correct statements is
   (a) $|a^2 - b^2| \le x \le |a^2 + b^2|$
   (b) $|a - b| \le x \le |a + b|$
   (c) $|a - b| < x < |a + b|$
   (d) $0 \le x \le |a - b|$ or $x \ge |a + b|$

2. A rod of length l leans by its upper end against a smooth vertical wall, while its other end leans against the floor. The end that leans against the wall moves uniformly downward. Then

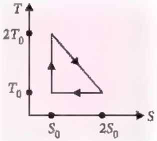


   (a) the other end also moves uniformly
   (b) the speed of other end goes on decreasing
   (c) the speed of other end goes on increasing
   (d) the speed of other end first decreases and then increases.

3. A blind person after walking 10 steps in one direction, each of length 80 cm, turns randomly to left or right. After walking 'n' steps, the maximum displacement of person is $16\sqrt{2}$ m. Then value of 'n' is
   (a) 20 (b) 30 (c) 40 (d) 60

4. For a monoatomic gas, work done at constant pressure is W. The heat supplied at constant volume for the same rise in temperature of the gas is
   (a) $W/2$ (b) $3W/2$ (c) $5W/2$ (d) W

5. The temperature-entropy diagram of a reversible engine cycle is given in figure. Its efficiency is



   (a) $\frac{2}{3}$
   (b) $\frac{1}{3}$
   (c) $\frac{1}{4}$
   (d) $\frac{1}{2}$

6. A cylinder of radius r and thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius r and outer radius $2r$, whose thermal conductivity is K_2. There is no loss of heat across cylindrical surfaces, when the ends of the combined system are maintained at temperatures T_1 and T_2. The effective thermal conductivity of the system, in the steady state is
   (a) $\frac{K_1K_2}{K_1+K_2}$ (b) $K_1 + K_2$
   (c) $\frac{K_1+3K_2}{4}$ (d) $\frac{3K_1+K_2}{4}$

7. Two moles of monoatomic gas is mixed with three moles of a diatomic gas. The molar specific heat of the mixture at constant volume is
   (a) 1.55 R (b) 2.10 R (c) 1.63 R (d) 2.20 R

8. A satellite with kinetic energy E is revolving round the earth in a circular orbit. The minimum additional kinetic energy required for it to escape into outer space is
   (a) $\sqrt{2}E$ (b) $2E$ (c) $E/2$ (d) E

9. Angle of a prism is A and its one surface is silvered. Light rays falling at an angle of incidence of $2A$ on first surface return back

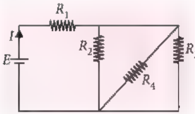
through the same path after suffering reflection at the second silvered surface. Refractive index of the material is

(a) 2 sin A (b) 2 cos A
(c) (1/2) cos A (d) tan A

10. In a hydrogen atom, an electron revolves with a frequency of 6.8×10^9 MHz in an orbit of diameter 1.06 Å. The equivalent magnetic moment is

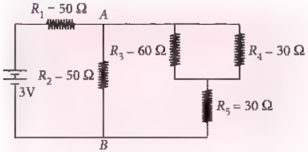
(a) 7.9×10^{-24} A m^2 (b) 9.7×10^{-24} A m^2
(c) 9.7×10^{24} A m^2 (d) 7.9×10^{24} A m^2

11. In the given circuit, E = 6.0 V, R_1 = 100 Ω, $R_2 = R_3$ = 50 Ω, R_4 = 75 Ω. The equivalent resistance of the circuit, in ohm is



(a) 11.875 (b) 26.31
(c) 118.75 (d) 16.39

12. In the given circuit, voltage across R_4 is

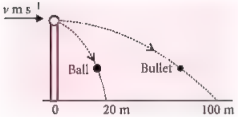


(a) 0.4 V (b) 0.6 V (c) 0.8 V (d) 1.0 V

13. The density of a cube is measured by measuring its mass and the length of its side. If the maximum errors in the measurements of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is

(a) 7% (b) 5% (c) 9% (d) 3%

14. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg travelling with a velocity v m s^{-1} in a horizontal direction, hits the centre of the ball. After the collision, the ball and the bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post.



The initial velocity v of the bullet is

(a) 250 m s^{-1} (b) $250\sqrt{2}$ m s^{-1}
(c) 400 m s^{-1} (d) 500 m s^{-1}

15. In Young's double slit experiment $\frac{d}{D} = 10^{-4}$ (d = distance between slits, D = distance of screen from the slits). At a point P on the screen the resulting intensity is equal to the intensity due to individual slit I_0. Then the distance of point P from the central maximum is

(Take λ = 6000 Å)

(a) 2 mm (b) 1 mm (c) 0.5 mm (d) 4 mm

16. A 12 V battery is applied in forward bias across a circuit having p-n junction and resistance R in series. A 0.6 V potential drop is across p-n junction and current is 2×10^{-3} A. The resistance R is

(a) 5.7×10^2 Ω (b) 5.7×10^3 Ω
(c) 5.7×10^4 Ω (d) 5.7×10^5 Ω

17. A block of mass 0.50 kg is moving with a speed of 2.00 m s^{-1} on a smooth surface. It strikes a stationary mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

18. A stone is hung in air from a wire which is stretched over a sono meter. The bridges of the sonometer are 40 cm apart when the wire is in unison with a tuning fork of frequency 256 Hz. When the stone is completely immersed in water, the length between the bridges is 22 cm for re-establishing unison. The specific gravity of the material of the stone is

(a) $\frac{(40)^2}{(40)^2+(22)^2}$ (b) $\frac{(40)^2}{(40)^2-(22)^2}$

(c) $256\times\frac{22}{40}$ (d) $256\times\frac{40}{22}$

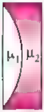
19. Two equal mass m and m are hung from a balance whose scale pans differ in vertical

height by 'h'. The error in weighing in terms of density of the earth ρ is

(a) $\pi G\rho mh$ (b) $\frac{1}{3}\pi G\rho mh$

(c) $\frac{8}{3}\pi G\rho mh$ (d) $\frac{4}{3}\pi G\rho mh$

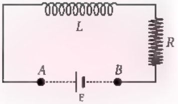
20. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If the lenses are made of different materials of refractive indices μ_1 and μ_2 and R is the radius of curvature of the curved surfaces of the lenses, then the focal length of the combination is



(a) $\frac{R}{(\mu_1-\mu_2)}$ (b) $\frac{2R}{(\mu_2-\mu_1)}$

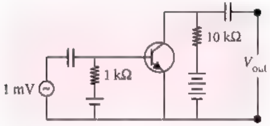
(c) $\frac{R}{2(\mu_1-\mu_2)}$ (d) $\frac{R}{2-(\mu_1+\mu_2)}$

21. An inductor (L = 100 mH), a resistor (R = 100 Ω) and a battery (ε = 100 V) are initially connected in series as shown in figure. After sometime, the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is



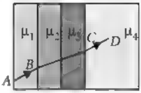
(a) e A (b) 0.1 A
(c) 1 A (d) $1/e$ A

22. In the common-emitter configuration, an n-p-n transistor with current gain 100 is used. The output voltage of the amplifier will be



(a) 10 mV (b) 0.1 V
(c) 1.0 V (d) 10 V

23. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the adjacent figure.



The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have

(a) $\mu_1 = \mu_3$ (b) $\mu_2 = \mu_4$
(c) $\mu_4 = \mu_1$ (d) $\mu_2 = \mu_3$

24. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(a) $\frac{l\omega}{6g}$ (b) $\frac{l^2\omega^2}{2g}$

(c) $\frac{l^2\omega^2}{6g}$ (d) $\frac{l^2\omega^2}{3g}$

25. $^{221}_{87}Ra$ is a radioactive substance having half-life of 4 days. The probability that a nucleus undergoes decay in two half lives is

(a) 1 (b) 1/2
(c) 3/4 (d) 1/4

26. An electron of mass m with an initial velocity $\vec{v} = v_0\hat{i}$ is in an electric field $\vec{E} = E_0\hat{j}$. If $\lambda_0 = h/mv_0$, its de Broglie wavelength at time t is given by

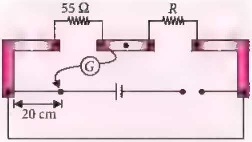
(a) λ_0 (b) $\lambda_0\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}$

(c) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$ (d) $\frac{\lambda_0}{\left(1+\frac{e^2E_0^2t^2}{m^2v_0^2}\right)}$

27. A proton and an α-particle are accelerated through the same potential difference. The ratio of de Broglie wavelength of proton to the de Broglie wavelength of alpha particle will be

(a) 1 : 2 (b) $2\sqrt{2}:1$
(c) 2 : 1 (d) 1 : 1

28. Figure shows a metre-bridge set up with null deflection in the galvanometer. The value of the unknown resistance R is



(a) 55 Ω (b) 13.75 Ω
(c) 220 Ω (d) 110 Ω

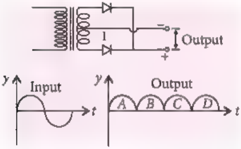
29. A short circuit coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(a) halved (b) $\frac{1}{4}^{\text{th}}$
(c) doubled (d) quadrupled

30. A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is

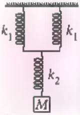
(a) 25% (b) 20% (c) 35% (d) 15%

31. A full-wave rectifier circuit along with the output is shown in figure. The contribution (s) from the diode 1 is (are)



(a) C (b) A, C
(c) B, D (d) A, B, C, D

32. What will be the force constant of the spring system shown in the figure?



(a) $\frac{k_1}{2}+k_2$

(b) $\left[\frac{1}{2k_1}+\frac{1}{k_2}\right]^{-1}$

(c) $\frac{1}{2k_1}+\frac{1}{k_2}$ (d) $\left[\frac{2}{k_1}+\frac{1}{k_2}\right]^{-1}$

33. If the wavelength of first line of Balmer series is 6563 Å, the wavelength of first line of Lyman series and Rydberg's constant respectively will be

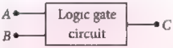
(a) 1215.4 Å, 1.1×10^7 m^{-1}
(b) 5863 Å, 2.0×10^7 m^{-1}
(c) 2316.4 Å, 0.1×10^7 m^{-1}
(d) none of these

34. In a photoemissive cell, with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be

(a) less than $v\left(\frac{4}{3}\right)^{1/2}$ (b) $v\left(\frac{4}{3}\right)^{1/2}$

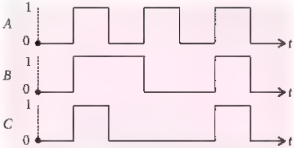
(c) $v\left(\frac{3}{4}\right)^{1/2}$ (d) greater than $v\left(\frac{4}{3}\right)^{1/2}$

35. Figure (a) shows a logic gate circuit with two inputs A and B and the output C.



(a)

The voltage waveforms of A, B and C are as shown in figure (b).

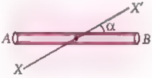


(b)

The logic gate is

(a) OR gate (b) AND gate
(c) NAND gate (d) NOR gate

36. The moment of inertia of a uniform rod of length $2l$ and mass m about an axis XX' passing through its centre and inclined at an angle α, as shown in figure is

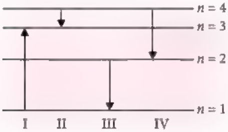


(a) $\frac{ml^2}{3}\sin^2\alpha$ (b) $\frac{ml^2}{12}\sin^2\alpha$

(c) $\frac{ml^2}{6}\cos^2\alpha$ (d) $\frac{ml^2}{2}\cos^2\alpha$

37. A metallic wire with tension T and at temperature 30°C vibrates with its fundamental frequency of 1 kHz. The same wire with the same tension but at 10°C temperature vibrates with a fundamental frequency of 1.001 kHz. The coefficient of linear expansion of the wire is
(a) $2 \times 10^{-4}\,°C^{-1}$ (b) $1.5 \times 10^{-4}\,°C^{-1}$
(c) $1 \times 10^{-4}\,°C^{-1}$ (d) $0.5 \times 10^{-4}\,°C^{-1}$

38. Figure shows the energy levels for an electron in a certain atom. Which of the following transitions shown, represents the emission of a photon with the most energy?



(a) III (b) IV (c) I (d) II

39. A parallel plate capacitor has the space between its plates filled by two slabs of thickness $\frac{d}{2}$ each and dielectric constant K_1 and K_2. d is the plate separation of the capacitor. The capacity of the capacitor is

(a) $\frac{2\varepsilon_0 d}{A}\left(\frac{K_1+K_2}{K_1K_2}\right)$ (b) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$

(c) $\frac{2\varepsilon_0 A}{A}(K_1+K_2)$ (d) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1+K_2}{K_1K_2}\right)$

40. The Poisson's ratio of a material is 0.5. If a force is applied to a wire of this material, there is a decrease in the cross-sectional area by 4%. The percentage increase in its length is
(a) 1% (b) 2% (c) 2.5% (d) 4%

41. The efficiency of a Carnot engine is 50% and temperature of sink is 500 K. If temperature of source is kept constant and its efficiency raised to 60%, then the required temperature of sink will be
(a) 100 K (b) 600 K (c) 400 K (d) 500 K

42. A simple pendulum with length L and mass m of the bob is vibrating with an amplitude a. Then the maximum tension in the string is

(a) mg (b) $mg\left[1+\left(\frac{a}{L}\right)^2\right]$

(c) $mg\left[1+\frac{a}{2L}\right]^2$ (d) $mg\left[1+\left(\frac{a}{L}\right)\right]^2$

43. Two rain drops reach the earth with different terminal velocities having ratio 9 : 4. Then the ratio of their volume is
(a) 3 : 2 (b) 4 : 9 (c) 9 : 4 (d) 27 : 8

44. A pendulum consists of a wooden bob of mass m and length l. A bullet of mass m_1 is fired towards the pendulum with a speed v_1. The bullet emerges out of the bob with a speed $v_1/3$, and the bob just completes the vertical circle. The value of v_1 is

(a) $\left(\frac{m}{m_1}\right)\sqrt{5gl}$ (b) $\frac{m}{m_1}\sqrt{\frac{g}{l}}$

(c) $\frac{2}{3}\left(\frac{m_1}{m}\right)\sqrt{5gl}$ (d) $\frac{3}{2}\left(\frac{m}{m_1}\right)\sqrt{5gl}$

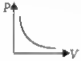
45. A hollow cylinder has a charge q C within it. If ϕ is the electric flux in unit of V m associated with the curved surface B, the flux linked with the plane surface A in unit of V m will be

(a) $\frac{1}{2}\left(\frac{q}{\varepsilon_0}-\phi\right)$ (b) $\frac{q}{2\varepsilon_0}$

(c) $\frac{\phi}{3}$ (d) $\frac{q}{\varepsilon_0}-\phi$

46. A force F is required to break a wire of length l and radius r. What force is required to break a wire, of the same material, having twice the length l and six times the radius r?
(a) F (b) $3F$ (c) $9F$ (d) $36F$

47. Which of the following graph best represents the variation of pressure P with volume V of a gas at fixed temperature?

(a)


(b)


(c)


(d)


48. The velocity of a body moving in viscous medium is given by $V = \frac{P}{Q}(1 - e^{Qt})$ where t is time; P and Q are constants. Then the dimensions of P are

(a) M^0LT^{-2} (b) $M^0L^2T^{-2}$
(c) $M^{-1}LT^{-2}$ (d) $M^0L^0T^{-2}$

49. If the atom ${}^{257}_{100}Fm$ follows the Bohr model and the radius of fifth orbit of ${}^{257}_{100}Fm$ is n times the Bohr radius, then find n.

(a) 100 (b) 200 (c) 4 (d) 1/4

50. An electric dipole is placed at an angle of 60° with an electric field of intensity 10^5 N C^{-1}. It experiences a torque equal to $8\sqrt{3}$ N m. Find the charge on the dipole, if the dipole length is 2 cm.

(a) 8×10^3 C (b) 8.54×10^{-4} C
(c) 8×10^{-3} C (d) 0.85×10^{-6} C

SOLUTIONS

1. **(b)** : Let accelerations of the particle, observer A and observer B be $\vec{p}$, $\vec{A}$, $\vec{B}$ m s ^{2} with respect to ground. Hence as per question

$$\vec{p}\quad \vec{A} = a\hat{n}_1 \qquad ...(i)$$

$$\vec{p} - \vec{B} = b\hat{n}_2 \qquad ...(ii)$$

Hence $\hat{n}_1$ and $\hat{n}_2$ are unit vectors depending upon direction of acceleration of the particle with respect to respective observers.

Subtracting equation (ii) and equation (i)

$$\vec{A} - \vec{B} = b\hat{n}_2 - a\hat{n}_1$$

$$\Rightarrow \quad |\vec{A} - \vec{B}| = |b\hat{n}_2 + a(-\hat{n}_1)|$$

$$\Rightarrow \quad x = \sqrt{b^2 + a^2 + 2ab\cos\theta}$$

Here θ is the angle between $\hat{n}_2$ and $-\hat{n}_1$

and $|\vec{A} - \vec{B}| = x$ (given)

Now,

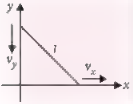
$$-1 \le \cos\theta \le 1$$

$$\therefore \quad |a - b| \le x \le |a + b|$$

2. **(b)** : From figure, $x^2 + y^2 = l^2$

$$\Rightarrow \quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{-yv_y}{x} = \frac{-yv_y}{\sqrt{l^2 - y^2}}$$

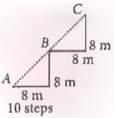


$$\Rightarrow \quad \left|\frac{dx}{dt}\right| = \frac{v_y}{\sqrt{l^2/y^2 - 1}}$$

Since y is decreasing, $\sqrt{l^2/y^2 - 1}$ is increasing continuously. Hence $\frac{v_y}{\sqrt{l^2/y^2 - 1}}$ is decreasing continuously.

3. **(c)**: The displacement will be maximum if he walks in the way as shown in figure. After walking 20 steps displacement is $8\sqrt{2}$ m.

$\therefore$ He walks 40 steps for displacement $16\sqrt{2}$ m.



4. **(b)** : For the process at constant pressure

$$dQ = C_p\,dT + dW \text{ or } dT = \frac{dQ - dW}{C_p}$$

For the process at constant volume,

$$dQ - C_v\,dT \qquad (\because dW - 0)$$

$$= C_v\left(\frac{dQ - dW}{C_p}\right) = \frac{dQ - dW}{C_p/C_v} = \frac{dQ - dW}{\gamma}$$

or $(\gamma - 1)dQ = dW$

$$\left(\frac{5}{3} - 1\right)dQ = W,\ dQ = \frac{3}{2}W.$$

5. **(b)**

6. **(c)**: $\left(\frac{\Delta Q}{\Delta t}\right)_{\text{inner}} + \left(\frac{\Delta Q}{\Delta t}\right)_{\text{outer}} = \left(\frac{\Delta Q}{\Delta t}\right)_{\text{total}}$

or $$\frac{K_1\pi r^2(T_2 - T_1)}{l} + \frac{K_2\pi[(2r)^2 - r^2](T_2 - T_1)}{l} = \frac{K\pi(2r)^2(T_2 - T_1)}{l}$$

or $$(K_1 + 3K_2)\frac{\pi r^2(T_2 - T_1)}{l} = \frac{K\pi 4r^2(T_2 - T_1)}{l}$$

$$\therefore \quad K = \frac{K_1 + 3K_2}{4}.$$

7. **(b)** : Molar specific heat of the mixture at constant volume is

$$C_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{(n_1 + n_2)} = \frac{2\left(\frac{3}{2}R\right) + 3\left(\frac{5}{2}R\right)}{2 + 3} = 2.10\,R$$

8. **(d)** : For a satellite revolving round the earth in circular orbit of radius r

$$\frac{mv_o^2}{r}=\frac{GMm}{r^2} \quad \text{or} \quad v_o=\sqrt{\frac{GM}{r}}$$

$$\therefore \quad E=\frac{1}{2}mv_o^2=\frac{1}{2}\frac{GMm}{r}$$

For the satellite to escape

$$\frac{1}{2}mv_e^2=\frac{GMm}{r} \qquad \text{or } v_e^2=\frac{2GM}{r}$$

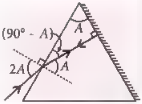
$$E_e=\frac{1}{2}mv_e^2=\frac{1}{2}\times\frac{m\times 2GM}{r}=\frac{mGM}{r}=2E$$

Total energy needed = $2E = E + E$

Extra energy needed = E.

9. **(b)** : Given : $i = 2A$

From figure $r = A$



$$\therefore \quad \mu=\frac{\sin i}{\sin r}$$

$$=\frac{\sin 2A}{\sin A}=2\cos A.$$

10. **(b)** : Here, $\upsilon = 6.8 \times 10^9$ MHz $= 6.8 \times 10^{15}$ Hz

$$r=\frac{1.06}{2}=0.53\ \text{Å}=0.53\times10^{-10}\ \text{m}$$

$$M=IA=\left(\frac{e}{1/\upsilon}\right)\pi r^2=e\upsilon\pi r^2$$

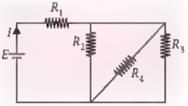
$$=(1.6\times10^{-19})(6.8\times10^{15})\times\frac{22}{7}(0.53\times10^{-10})^2$$

$$=9.7\times10^{-24}\ \text{A m}^2$$

11. **(c)**: In the given circuit, three resistances R_2, R_4 and R_3 are in parallel, hence,

$$\frac{1}{R}=\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_3}$$

$$=\frac{1}{50}+\frac{1}{50}+\frac{1}{75}$$



$$=\frac{75+75+50}{50\times75}$$

$$\therefore \quad R=\frac{50\times75}{200}=18.75\ \Omega$$

This resistance is in series with R_1.

Hence, equivalent resistance of the circuit

$= R_1 + R = 100 + 18.75$

$= 118.75\ \Omega$.

12. **(a)**

13. **(c)**: $\because \quad \rho=\frac{M}{L^3}$,

$$\therefore \quad \frac{\Delta\rho}{\rho}=\frac{\Delta M}{M}+3\frac{\Delta L}{L}=3\%+3(2\%)=9\%.$$

14. **(d)** : As $y=\frac{1}{2}gt^2$,

$$\therefore 5=\frac{1}{2}(10)t^2 \quad \text{or} \quad t=1\ \text{s}$$

Further, as $(v_{\text{ball}})t = 20$ m,

$$v_{\text{ball}}=\frac{20\ \text{m}}{1\ \text{s}}=20\ \text{m s}^{-1}$$

and as $(v_{\text{bullet}})t = 100$ m,

$$v_{\text{bullet}}=\frac{100\ \text{m}}{1\ \text{s}}=100\ \text{m s}^{-1}$$

Applying the law of conservation of momentum,

$(0.01\ \text{kg})v = (0.01\ \text{kg})\ (100\ \text{m s}^{-1})$

$+ (0.2\ \text{kg})\ (20\ \text{m s}^{-1})$.

which givensv $v = 500$ m s^{-1}.

15. **(a)** : We know that $I=4I_0\cos^2\left(\frac{\phi}{2}\right)$

$$I_0=4I_0\cos^2\left(\frac{\phi}{2}\right) \qquad (\because\ I=I_0\ \text{(given)})$$

$$\text{or} \quad \cos\left(\frac{\phi}{2}\right)=\frac{1}{2} \quad \text{or} \quad \frac{\phi}{2}=\frac{\pi}{3}$$

$$\text{or} \quad \phi=\frac{2\pi}{3}=\frac{2\pi}{\lambda}\ \Delta x \ \text{ or } \ \frac{1}{3}=\frac{1}{\lambda}\left(\frac{yd}{D}\right)$$

$$\therefore \quad y=\frac{\lambda}{3\left(\frac{d}{D}\right)}=\frac{6\times10^{-7}}{3\times10^{-4}}=2\times10^{-3}\ \text{m}=2\ \text{mm}.$$

16. **(b)** : $R=\frac{V-V_D}{I}=\frac{12\ \text{V}-0.6\ \text{V}}{2\times10^{-3}\ \text{A}}=5.7\times10^3\ \Omega.$

17. **(c)**: Here, $m_1 = 0.50$ kg, $u_1 = 2.00$ m s^{-1}

$m_2 = 1.00$ kg, $u_2 = 0$

If v is velocity of the combination after collision, then according to the principle of conservation of linear momentum,

$(m_1 + m_2)v = m_1u_1 + m_2u_2 = m_1u_1$

$$v=\frac{m_1u_1}{m_1+m_2}=\frac{0.50\times2.00}{0.50+1.00}$$

$$v=\frac{1.00}{1.50}=\frac{2}{3}\ \text{m s}^{-1}$$

Energy loss = initial energy – final energy

$$=\frac{1}{2}m_1u_1^2-\frac{1}{2}(m_1+m_2)v^2$$

$$=\frac{1}{2}\times0.50\times(2.00)^2-\frac{1}{2}(0.50+1.00)\left(\frac{2}{3}\right)^2$$

$$= 1.00 - \frac{1.50}{2} \times \frac{4}{9} = 0.67 \text{ J}.$$

18. (b)

19. (c): As $g_h = \frac{g}{\left[1+\frac{h}{R}\right]^2} = g\left[1-\frac{2h}{R}\right]$

$\therefore$ error in weighing $= W_2 - W_1 = mg_2 - mg_1$

$$= 2mg\left[\frac{h_1}{R} - \frac{h_2}{R}\right] = 2m\frac{GM}{R^2}\frac{h}{R}$$

$$\left[\because\ g = \frac{GM}{R^2} \text{ and } h_1 - h_2 = h\right]$$

$$= 2mG \cdot \frac{4}{3}\pi R^3 \rho \frac{h}{R^3} = \frac{8\pi}{3} Gm\rho h.$$

20. (a) : As $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\therefore \quad \frac{1}{F} = (\mu_1 - 1)\left(\frac{1}{\infty} + \frac{1}{R}\right) + (\mu_2 - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right)$$

$$= \frac{\mu_1 - \mu_2}{R}$$

or $\quad F = \frac{R}{(\mu_1 - \mu_2)}$

21. (d) : Here, $L = 100$ mH $= 100 \times 10^{-3}$ H $= 0.1$ H
$R = 100\ \Omega$, $\varepsilon = 100$ V

$$I_0 = \frac{\varepsilon}{R} = \frac{100}{100} = 1 \text{ A}$$

When the short circuiting is done and battery is disconnected, the current in RL circuit decays. At any time t,

$$I = I_0 e^{\frac{-R}{L}t}$$

At $t = 1$ ms, $I = 1 \times e^{\frac{-100}{0.1}\times 10^{-3}} = e^{-1} = \frac{1}{e}$ A.

22. (c): As $A_v = \frac{V_0}{V_i} = \beta\left(\frac{R_{out}}{R_{in}}\right)$;

$$V_0 = V_i \beta\left(\frac{R_{out}}{R_{in}}\right)$$

Here, $V_i = 1$ mV $= 1 \times 10^{-3}$ V, $\beta = 100$
$R_{out} = 10$ kΩ, $R_{in} = 1$ kΩ

$$\therefore \quad V_0 = (1\times 10^{-3}\text{ V})(100)\left(\frac{10\text{ k}\Omega}{1\text{ k}\Omega}\right) = 1 \text{ V}.$$

23. (c)

24. (c): If centre of mass rises to maximum height h, then loss in K.E. = gain in P.E.,

$$\frac{1}{2}I\omega^2 = mgh \quad \text{or} \quad \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = mgh$$

$$\therefore \quad h = \frac{l^2\omega^2}{6g}.$$

25. (c): As $\frac{N_0}{N} = 2^{t/T_{1/2}} = 2^2 = 4 \quad (\because\ t = 2T_{1/2})$

$$\therefore \quad N = \frac{N_0}{4}$$

The sample that decays is $N_0 - \frac{N_0}{4} = \frac{3N_0}{4}$

$$\therefore \quad \text{Required probability} = \frac{3N_0/4}{N_0} = \frac{3}{4}$$

26. (c)

27. (b) : As $\lambda = \frac{h}{\sqrt{2mqV}} \quad \therefore \quad \frac{\lambda_p}{\lambda_\alpha}$

$$= \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p(2e)}{m_p e}} = 2\sqrt{2} \text{ or } \lambda_p : \lambda_\alpha = 2\sqrt{2} : 1$$

28. (c): As bridge is balanced, so $\frac{55}{R} = \frac{20}{80} = \frac{1}{4}$

or $\quad R = 55 \times 4 = 220\ \Omega$

29. (b) : Power dissipated, $P = \frac{\varepsilon^2}{R}$, where

induced emf $\varepsilon = -\left(\frac{d\phi}{dt}\right)$ and $\phi = NBA$

$$\therefore \quad \varepsilon = -NA\left(\frac{dB}{dt}\right)$$

Also $R \propto \frac{l}{a} \propto \frac{l}{r^2}$

$$\therefore \quad P = \frac{\varepsilon^2}{R} \propto \frac{N^2A^2r^2}{l} \propto \frac{N^2r^6}{l}$$

As r is halved and N is quadrupled,
P becomes $1/4^{th}$.

30. (b) : Let total length of the chain = L and maximum length of the chain that can hang = l
$\therefore$ Length of the chain on the table = $(L - l)$.
If ρ is the mass per unit length of the chain, weight of the chain on the table, $W = (L - l)\rho g$
Force acting on this length due to hanging part of the chain = weight of the hanging part of the chain of length $l = l\rho g$.

This obviously is the force of friction (f).

As $\mu = \frac{f}{R} = \frac{l\rho g}{(L-l)\rho g} = \frac{l}{L-l}$

[as $R = W = (L-l)\rho g$]

$\frac{l}{L-l} = 0.25 = \frac{1}{4}$ or $l = \frac{L}{5} = 20\%$

31. (c) : During positive half cycle of ac input, upper diode (forward biased) conducts and lower diode (*i.e.*, 1) which is reverse-biased does not conduct. During the negative half of ac input, reverse is the case and the lower diode conducts.

32. (b) : For parallel combination of first two identical springs of spring constant k_1, effective spring constant, $k_p = 2k_1$

Now, springs of spring constants k_p and k_2 are joined in series, so the force constant or the spring constant of the system is

$$\frac{1}{k_s} = \frac{1}{k_p} + \frac{1}{k_2}$$

$$\therefore \quad k_s = \left[\frac{1}{k_p} + \frac{1}{k_2}\right]^{-1} = \left[\frac{1}{2k_1} + \frac{1}{k_2}\right]^{-1}$$

33. (a) : As for the first line of Balmer series,

$$\frac{1}{\lambda_B} = \frac{5R}{36} \quad \text{...(i)}$$

and for the first line of Lyman series,

$$\frac{1}{\lambda_L} = \frac{3R}{4} \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$$\lambda_L = \frac{5}{27}\lambda_B = \frac{5}{27} \times 6563\ \text{Å} = 1215.4\ \text{Å}$$

Now,

$$R = \frac{4}{3\lambda_L} = \frac{4}{3(1215.4 \times 10^{-10}\ \text{m})} \approx 1.1 \times 10^7\ \text{m}^{-1}$$

34. (d) : From Einstein's photoelectric equation, we get

$$\frac{hc}{\lambda} = h\nu_0 + \frac{1}{2}mv^2 \quad \text{...(i)}$$

and $\frac{hc}{3\lambda/4} = h\nu_0 + \frac{1}{2}mv'^2$,

or $\frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{hc}{\lambda}\right) - h\nu_0 = \frac{4}{3}\left(h\nu_0 + \frac{1}{2}mv^2\right) - h\nu_0$ ...(Using (i))

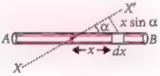
$$\frac{1}{3}h\nu_0 + \frac{4}{3}\left(\frac{1}{2}mv^2\right)$$

or $\frac{1}{2}mv'^2 > \frac{4}{3}\left(\frac{1}{2}mv^2\right)$ or $v'^2 > \frac{4}{3}v^2$

or $v' > v\left(\frac{4}{3}\right)^{1/2}$

35. (b) : The logic gate is an AND gate as output C is high only when both inputs A and B are high.

36. (a) : Mass of small element of length dx of the rod at a distance x from the centre $dm = \left(\frac{m}{2l}\right)dx$.



Perpendicular distance of this element from the axis $XX' = x \sin a$,

$\therefore$ moment of inertia of this element,

$$dI = \left(\frac{m}{2l}\right)dx\,(x\sin\alpha)^2$$

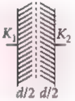
$$I = \int_{x=-l}^{x=+l}\left(\frac{m}{2l}\right)dx\,(x\sin\alpha)^2$$

$$= \frac{m}{2l}\sin^2\alpha\left(\frac{x^3}{3}\right)_{x=-l}^{x=+l}$$

$$I = \frac{m}{2l}\sin^2\alpha\left(\frac{2l^3}{3}\right) = \frac{ml^2}{3}\sin^2\alpha$$

37. (d) 38. (a)

39. (b) : The capacities of two individual capacitors is $C_1 = \frac{K_1\varepsilon_0 A}{d/2} = \frac{2K_1\varepsilon_0 A}{d}$



and $C_2 = \frac{2K_2\varepsilon_0 A}{d}$

Now, C_1 and C_2 are in series,

$$\therefore \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\varepsilon_0 A} + \frac{d}{2K_2\varepsilon_0 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1+K_2}{K_1K_2}\right) \text{ or } C_s = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right).$$

40. (d)

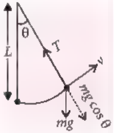
41. (c): As $\eta = 1 - \frac{T_2}{T_1}$

$\therefore \quad \frac{50}{100} = 1 - \frac{500}{T_1}$ or $T_1 = 1000\text{ K}$

Again, $\frac{60}{100} = 1 - \frac{T_2'}{1000}$

or $T_2' = 400\text{ K}$

42. (b) : $T - mg\cos\theta = \frac{mv^2}{L}$



T is maximum, when $\theta = 0°$.

$\therefore T_{max} = mg + \frac{mv_{max}^2}{L}$

$= mg + \frac{ma^2\omega^2}{L}$

$= mg + \frac{ma^2}{L} \times \frac{g}{L} = mg\left[1 + \left(\frac{a}{L}\right)^2\right]$

43. (d) : As terminal velocity,

$v = \frac{2}{9}\frac{(\rho - \sigma)g}{\eta} r^2$, *i.e.*, $v \propto r^2 \quad \therefore \quad \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2$

or $\frac{9}{4} = \left(\frac{r_1}{r_2}\right)^2$ or $\frac{r_1}{r_2} = \frac{3}{2}$

$$\frac{(\text{Volume})_1}{(\text{Volume})_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

44. (d) : For completing a vertical circle, velocity of the bob

$v = \sqrt{5gl}$ ...(i)

Using principle of conservation of linear momentum,

$mv = m_1\left(\frac{v_1 - v_1}{3}\right) = m_1\left(\frac{2}{3}v_1\right)$

or $v = \frac{2}{3}\frac{m_1}{m}v_1$ ...(ii)

From (i) and (ii),

$\frac{2}{3}\frac{m_1}{m}v_1 = \sqrt{5gl}$ or $v_1 = \frac{3}{2}\frac{m}{m_1}\sqrt{5gl}$

45. (a) : Gauss's law states that the net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0. *i.e.*,

$\phi_{total} = \frac{q}{\varepsilon_0}$.

Let electric flux linked with surfaces A, B and C are ϕ_A, ϕ_B and ϕ_C respectively.

That is $\phi_{total} = \phi_A + \phi_B + \phi_C$

Since $\phi_C = \phi_A$,

$\therefore \quad 2\phi_A + \phi_B = \phi_{total} = \frac{q}{\varepsilon_0}$

or $\phi_A = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi_B\right)$

$\phi_A = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right) \qquad (\because \phi_B = \phi)$

46. (d) : Breaking force does not depend upon length.

Breaking force = breaking stress × area of cross-section.

For a given material, breaking stress is constant.

$\therefore$ Breaking force $\propto$ area of cross-section

$\therefore \quad \frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{\pi(6r)^2}{\pi r^2} = 36$

or $F_2 = 36F_1 = 36\,F$

47. (a) : The relation of P and V when temperature is constant, is given by Boyle's law as PV = constant = k or $P = k/V$. The variation of P and V is as shown in option (a).

48. (a) : $V = \frac{P}{Q}(1 - e^{Qt})$

As Qt = number, $\therefore \quad [Q] = \frac{1}{[t]} = \text{T}^{-1}$

Now, $\frac{P}{Q} = V$,

$\therefore \quad [P] = [Q \times V] = \text{T}^{-1} \times \text{LT}^{-1} = [\text{M}^0\text{LT}^{-2}]$

49. (d)

50. (c): Here, $\theta = 60°$, $E = 10^5\text{ N C}^{-1}$

$\tau = 8\sqrt{3}\text{ N m}$, $2a = 2\text{ cm} = 2 \times 10^{-2}\text{ m}$

From $\tau = pE\sin\theta = q(2a)E\sin\theta$

$$q = \frac{\tau}{2aE\sin\theta} = \frac{8\sqrt{3}}{2 \times 10^{-2} \times 10^5 \times \sin 60°}$$

$$= \frac{8\sqrt{3}}{2 \times 10^3 \times \sqrt{3}/2}$$

or $\quad q = 8 \times 10^{-3}\text{ C}$.

SOLVED PAPER 2014

Kerala PET

1. In a simple pendulum experiment, the maximum percentage error in the measurement of length is 2% and that in the observation of the time-period is 3%. Then the maximum percentage error in determination of the acceleration due to gravity g is
(a) 5% (b) 6% (c) 7% (d) 8%
(e) 10%

2. The pitch and the number of circular scale divisions in a screw gauge with least count 0.02 mm are respectively
(a) 1 mm and 100 (b) 0.5 mm and 50
(c) 1 mm and 50 (d) 0.5 mm and 100
(e) 1 mm and 200

3. A ball is dropped from the top of a tower of height 100 m and at the same time another ball is projected vertically upwards from ground with a velocity 25 m s^{-1}. Then the distance from the top of the tower, at which the two balls meet is
(a) 68.4 m (b) 48.4 m
(c) 18.4 m (d) 28.4 m
(e) 78.4 m

4. The ratio of distances traversed in successive intervals of time when a body falls freely under gravity from certain height is
(a) 1 : 2 : 3 (b) 1 : 5 : 9
(c) 1 : 3 : 5 (d) $\sqrt{1}:\sqrt{2}:\sqrt{3}$
(e) 1 : 4 : 9

5. A particle starting with certain initial velocity and uniform acceleration covers a distance of 12 m in first 3 seconds and a distance of 30 m in next 3 seconds. The initial velocity of the particle is
(a) 3 m s^{-1} (b) 2.5 m s^{-1}
(c) 2 m s^{-1} (d) 1.5 m s^{-1}
(e) 1 m s^{-1}

6. A ball of mass 10 g moving perpendicular to the plane of the wall strikes it and rebounds in the same line with the same velocity. If the impulse experienced by the wall is 0.54 N s, the velocity of the ball is
(a) 27 m s^{-1} (b) 3.7 m s^{-1}
(c) 54 m s^{-1} (d) 37 m s^{-1}
(e) 5.4 m s^{-1}

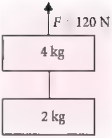
7. A particle has the position vector $\vec{r}=\hat{i}-2\hat{j}+\hat{k}$ and the linear momentum $\vec{p}=2\hat{i}-\hat{j}+\hat{k}$. Its angular momentum about the origin is
(a) $-\hat{i}+\hat{j}-3\hat{k}$ (b) $-\hat{i}+\hat{j}+3\hat{k}$
(c) $\hat{i}-\hat{j}+3\hat{k}$ (d) $\hat{i}-\hat{j}-5\hat{k}$
(e) $\hat{i}-\hat{j}+5\hat{k}$

8. The vertical component of velocity of a projectile at its maximum height (u - velocity of projection, θ - angle of projection) is
(a) $u\sin\theta$ (b) $u\cos\theta$
(c) $\dfrac{u}{\sin\theta}$ (d) 0
(e) $\dfrac{u}{\cos\theta}$

9. The coordinates of a particle moving in x-y plane at any instant of time t are $x=4t^2$; $y=3t^2$. The speed of the particle at that instant is
(a) $10t$ (b) $5t$ (c) $3t$ (d) $2t$
(e) $\sqrt{13}t$

10. A cyclist bends while taking turn in order to
(a) reduce friction
(b) provide required centripetal force
(c) reduce apparent weight
(d) reduce speed
(e) sit comfortably

11. Two blocks of masses 2 kg and 4 kg are attached by an inextensible light string as shown in the figure. If a force of 120 N pulls the blocks vertically upward, the tension in the string is (Take $g = 10$ m s^{-2})



(a) 20 N (b) 15 N (c) 35 N (d) 40 N
(e) 30 N

12. The total energy of a solid sphere of mass 300 g which rolls without slipping with a constant velocity of 5 m s^{-1} along a straight line is
(a) 5.25 J (b) 3.25 J (c) 0.25 J (d) 1.25 J
(e) 0.625 J

13. A bullet when fired into a target loses half of its velocity after penetrating 20 cm. Further distance of penetration before it comes to rest is
(a) 6.66 cm (b) 3.33 cm
(c) 12.5 cm (d) 10 cm
(e) 5 cm

14. In elastic collision
(a) both momentum and kinetic energy are conserved.
(b) neither momentum nor kinetic energy is conserved
(c) only momentum is conserved.
(d) only kinetic energy is conserved.
(e) forces involved in the interaction are non-conservative.

15. Two discs rotating about their respective axis of rotation with angular speeds 2 rad s^{-1} and 5 rad s^{-1} are brought into contact such that their axes of rotation coincide. Now, the angular speed of the system becomes 4 rad s^{-1}. If the moment of inertia of the second disc is 1×10^{-3} kg m^2, then the moment of inertia of the first disc (in kg m^2) is
(a) 0.25×10^{-3} (b) 1.5×10^{-3}
(c) 1.25×10^{-3} (d) 0.75×10^{-3}
(e) 0.5×10^{-3}

16. A wheel is rotating at 1800 rpm about its own axis. When the power is switched off, it comes to rest in 2 minutes. Then the angular retardation in rad s^{-1} is
(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$
(e) $\frac{\pi}{6}$

17. If the angular momentum of a particle of mass m rotating along a circular path of radius r with uniform speed is L, the centripetal force acting on the particle is
(a) $\frac{L^2}{mr^3}$ (b) $\frac{L^2}{mr}$
(c) $\frac{L}{mr^2}$ (d) $\frac{L^2 m}{r}$
(e) $\frac{Lm}{r^2}$

18. Pick out the wrong statement from the following.
(a) The SI unit of universal gravitational constant is N m^2 kg^{-2}.
(b) The gravitational force is a conservative force.
(c) The force of attraction due to a hollow spherical shell of uniform density on a point mass inside it is zero.
(d) The centripetal acceleration of the satellite is equal to acceleration due to gravity.
(e) Gravitational potential energy $= \frac{\text{gravitation potential}}{\text{mass of the body}}$.

19. If a body of mass m has to be taken from the surface of earth to a height $h = R$, then the amount of energy required is (R : radius of earth)
(a) mgR (b) $\frac{mgR}{3}$ (c) $\frac{mgR}{2}$ (d) $\frac{mgR}{12}$
(e) $\frac{mgR}{9}$

20. The total energy of an artificial satellite of mass m revolving in a circular orbit around the earth with a speed v is
(a) $\frac{1}{2}mv^2$ (b) $\frac{1}{4}mv^2$

(c) $-\frac{1}{4}mv^2$ (d) $-mv^2$

(e) $-\frac{1}{2}mv^2$

21. Two soap bubbles each with radius r_1 and r_2 coalesce in vacuum under isothermal conditions to form a bigger bubble of radius R. Then R is equal to

(a) $\sqrt{r_1^2 + r_2^2}$ (b) $\sqrt{r_1^2 - r_2^2}$

(c) $r_1 + r_2$ (d) $\frac{\sqrt{r_1^2 + r_2^2}}{2}$

(e) $2\sqrt{r_1^2 + r_2^2}$

22. The ratio of hydraulic stress to the corresponding strain is known as

(a) compressibility (b) bulk modulus
(c) Young's modulus (d) rigidity modulus
(e) expansion coefficient

23. A boy can reduce the pressure in his lungs to 750 mm of mercury. Using a straw he can drink water from a glass upto the maximum depth of (atmospheric pressure = 760 mm of mercury, density of mercury = 13.6 g cm^{-3})

(a) 13.6 cm (b) 9.8 cm
(c) 10 cm (d) 76 cm
(e) 1.36 cm

24. A spring stores 1 J of energy for a compression of 1 mm. The additional work to be done to compress it further by 1 mm is

(a) 1 J (b) 2 J (c) 3 J (d) 4 J
(e) 0.5 J

25. If m represents the mass of each molecule of a gas and T, its absolute temperature, then the root mean square velocity of the gaseous molecule is proportional to

(a) mT (b) $m^{1/2}\,T^{1/2}$
(c) $m^{-1/2}\,T$ (d) $m^{-1/2}\,T^{1/2}$
(e) $mT^{-1/2}$

26. A Carnot engine operating between temperatures T_1 and T_2 has efficiency 0.2. When T_2 is reduced by 50 K, its efficiency increases to 0.4. Then T_1 and T_2 are respectively

(a) 200 K, 150 K (b) 250 K, 200 K
(c) 300 K, 250 K (d) 300 K, 200 K
(e) 300 K, 150 K

27. A molecule of a gas has six degrees of freedom. Then the molar specific heat of the gas at constant volume is

(a) $\frac{R}{2}$ (b) R (c) $\frac{3R}{2}$ (d) $2R$
(e) $3R$

28. Total number of degrees of freedom of a rigid diatomic molecule is

(a) 3 (b) 6 (c) 5 (d) 2
(e) 7

29. If the differential equation for a simple harmonic motion is $\frac{d^2y}{dt^2} + 2y = 0$, the time-period of the motion is

(a) $\pi\sqrt{2}$ s (b) $\frac{\sqrt{2}}{\pi}$ s

(c) $\frac{\pi}{\sqrt{2}}$ s (d) 2π s

(e) $\frac{\sqrt{\pi}}{2}$ s

30. Identify the wrong statement from the following.

(a) If the length of a spring is halved, the time period of each part becomes $\frac{1}{\sqrt{2}}$ times the original.
(b) The effective spring constant K of springs in parallel is given by $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \ldots$
(c) The time period of a stiffer spring is less than that of a soft spring.
(d) The spring constant is inversely proportional to the spring length.
(e) The unit of spring constant is N m^{-1}.

31. The total energy of the particle executing simple harmonic motion of amplitude A is 100 J. At a distance of $0.707A$ from the mean position, its kinetic energy is

(a) 25 J (b) 50 J (c) 100 J (d) 12.5 J
(e) 70 J

32. Two travelling waves, $y_1 = A\sin[k(x + ct)]$ and $y_2 = A\sin[k(x - ct)]$ are superposed on a string. The distance between adjacent antinodes is

(a) $\frac{ct}{\pi}$ (b) $\frac{ct}{2\pi}$ (c) $\frac{\pi}{2k}$ (d) $\frac{k}{\pi}$

(e) $\frac{\pi}{k}$

33. If a stretched wire is vibrating in the second overtone, then the number of nodes and antinodes between the ends of the string are respectively

(a) 2 and 2 (b) 1 and 2
(c) 4 and 3 (d) 2 and 3
(e) 3 and 4

34. Pick out the correct statement in the following with reference to stationary wave pattern.

(a) In a tube closed at one end, all the harmonics are present.
(b) In a tube open at one end, only even harmonics are present.
(c) The distance between successive nodes is equal to the wavelength.
(d) In a stretched string, the first overtone is the same as the second harmonic.
(e) Reflection of a wave from a rigid wall changes the phase by 45°.

35. A plane square sheet of charge of side 0.5 m has uniform surface charge density. An electron at 1 cm from the centre of the sheet experiences a force of 1.6×10^{-12} N directed away from the sheet. The total charge on the plane square sheet is ($\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 m^{-2} N^{-1})

(a) 16.25 μC (b) −22.15 μC
(c) −44.27 μC (d) 144.27 μC
(e) 8.854 μC

36. The energy stored in a capacitor of capacitance C having a charge Q under a potential V is

(a) $\frac{1}{2}Q^2V$ (b) $\frac{1}{2}C^2V$

(c) $\frac{1}{2}\frac{Q^2}{V}$ (d) $\frac{1}{2}QV$

(e) $\frac{1}{2}CV$

37. The electrostatic force between two point charges is directly proportional to the

(a) sum of the charges
(b) distance between the charges
(c) permittivity of the medium
(d) square of the distance between the charges
(e) product of the charges

38. The time period of revolution of a charge q_1 and of mass m moving in a circular path of radius r due to Coulomb force of attraction with another charge q_2 at its centre is

(a) $\sqrt{\frac{16\pi\varepsilon_0 mr^3}{q_1q_2}}$ (b) $\sqrt{\frac{8\pi^2\varepsilon_0 mr^3}{q_1q_2}}$

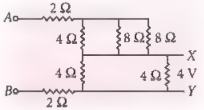
(c) $\sqrt{\frac{\varepsilon_0 mr^3}{16q_1q_2}}$ (d) $\sqrt{\frac{16\pi^3\varepsilon_0 mr^3}{q_1q_2}}$

(e) $\sqrt{\frac{\pi^2\varepsilon_0 mr^3}{8q_1q_2}}$

39. A point charge of 2 C experiences a constant force of 1000 N when moved between two points separated by a distance of 2 cm in a uniform electric field. The potential difference between the two points is

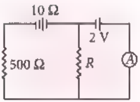
(a) 12 V (b) 8 V (c) 10 V (d) 16 V
(e) 5 V

40. In the network shown below, if potential across XY is 4 V, then the input potential across AB is



(a) 16 V (b) 20 V (c) 8 V (d) 12 V
(e) 24 V

41. If the ammeter A shows a zero reading in the circuit shown below, the value of resistance R is



(a) 500 Ω (b) 125 Ω (c) 100 Ω (d) 41.5 Ω
(e) 4 Ω

42. Five cells each of emf E and internal resistance r send the same amount of current through an external resistance R whether the cells are connected in parallel or in series. Then the ratio $\left(\frac{R}{r}\right)$ is

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) 1
(e) 5

43. The power dissipated in the transmission cables carrying current I and voltage V is inversely proportional to

(a) V (b) V^2 (c) $\sqrt{V}$ (d) $\sqrt{I}$
(e) I

44. A rigid container with thermally insulated walls contains a gas and a coil of resistance 50 Ω, carrying a current of 1 A. The change in internal energy of the gas after 2 minutes will be

(a) 6 kJ (b) 10 kJ (c) 3 kJ (d) 12 kJ
(e) 1.5 kJ

45. The magnitude of the magnetic field inside a long solenoid is increased by

(a) decreasing its radius
(b) decreasing the current through it
(c) increasing its area of cross-section
(d) introducing a medium of higher permeability
(e) decreasing the number of turns in it

46. A bar magnet of moment of inertia 9×10^{-5} kg m^2 placed in a vibration magnetometer and oscillating in a uniform magnetic field $16\pi^2 \times 10^{-5}$ T makes 20 oscillations in 15 s. The magnetic moment of the bar magnet is

(a) 3 A m^2 (b) 2 A m^2
(c) 5 A m^2 (d) 6 A m^2
(e) 4 A m^2

47. Identify the correctly matched pair.

	Material		Example
(a)	Diamagnetic	-	Gadolinium
(b)	Soft ferromagnetic	-	Alnico
(c)	Hard ferromagnetic	-	Copper
(d)	Paramagnetic	-	Sodium
(e)	Permanent magnet	-	Aluminium

48. If the radius of the dees of cyclotron is r, then the kinetic energy of a proton of mass m accelerated by the cyclotron at an oscillating frequency υ is

(a) $4\pi^2 m^2 \upsilon^2 r^2$ (b) $4\pi^2 m \upsilon^2 r^2$
(c) $2\pi^2 m \upsilon^2 r^2$ (d) $\pi^2 m \upsilon^2 r^2$
(e) $\pi^2 m^2 \upsilon^2 r^2$

49. If a magnetic dipole of moment M situated in the direction of a magnetic field B is rotated by 180°, then the amount of work done is

(a) MB (b) $2MB$ (c) $\frac{MB}{\sqrt{2}}$ (d) 0
(e) $\sqrt{MB}$

50. The polarity of induced emf is given by

(a) Ampere's circuital law
(b) Biot Savart law
(c) Lenz's law
(d) Fleming's right hand rule
(e) Fleming's left hand rule

51. In an LCR series circuit, at resonance

(a) the current and voltage are in phase
(b) the impedance is maximum
(c) the current is minimum
(d) the quality factor is independent of R
(e) the current leads the voltage by $\frac{\pi}{2}$

52. A conducting ring of radius 1 m kept in a uniform magnetic field B of 0.01 T, rotates uniformly with an angular velocity 100 rad s^{-1} with its axis of rotation perpendicular to B. The maximum induced emf in it is

(a) 1.5π V (b) π V
(c) 2π V (d) 0.5π V
(e) 4π V

53. A step down transformer increases the input current 4 A to 24 A at the secondary. If the number of turns in the primary coil is 330, the number of turns in the secondary coil is

(a) 60 (b) 50 (c) 65 (d) 45
(e) 55

54. In a plane electromagnetic wave, the electric field of amplitude 1 V m^{-1} varies with time in free space. The average energy density of magnetic field is (in J m^{-2})
(a) 8.86×10^{-12} (b) 4.43×10^{-12}
(c) 17.72×10^{-12} (d) 2.21×10^{-12}
(e) 1.11×10^{-12}

55. Which one of the following is the property of a monochromatic, plane electromagnetic wave in free space?
(a) Electric and magnetic fields have a phase difference of $\pi/2$.
(b) The energy contribution of both electric and magnetic fields are equal.
(c) The direction of propagation is in the direction of electric field E.
(d) The pressure exerted by the wave is the product of energy density and the speed of the wave.
(e) The speed of the wave is B/E

56. The apparent flattening of the sun at sunset and sunrise is due to
(a) refraction (b) diffraction
(c) total internal reflection
(d) interference (d) polarisation

57. The polarising angle for a medium is found to be 60°. The critical angle of the medium is
(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{4}\right)$
(e) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

58. Identify the mismatch in the following.
(a) Myopia - Concave lens
(b) For rear view - Concave mirror
(c) Hypermetropia - Convex lens
(d) Astigmatism - Cylindrical lens
(e) Reflecting telescope - Convex mirror

59. In Young's double slit experiment, to increase the fringe width
(a) the wavelength of the source is increased
(b) the source is moved towards the slit
(c) the source is moved away from the slit
(d) the slit separation is increased
(e) the screen is moved towards the slit

60. Light of wavelength 5000 Å is incident normally on a slit of width 2.5×10^{-4} cm. The angular position of second minimum from the central maximum is
(a) $\sin^{-1}\left(\frac{1}{5}\right)$ (b) $\sin^{-1}\left(\frac{2}{5}\right)$
(c) $\left(\frac{\pi}{3}\right)$ (d) $\left(\frac{\pi}{6}\right)$
(e) $\left(\frac{\pi}{4}\right)$

61. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential. Then the ratio of their de Broglie wavelengths is
(a) 1 (b) $\sqrt{\frac{m_e}{m_p}}$ (c) $\frac{m_e}{m_p}$ (d) $\frac{m_p}{m_e}$
(e) $\sqrt{\frac{m_p}{m_e}}$

62. The half-life of a radioactive substance is 20 minutes. The time taken between 50% decay and 87.5% decay of the substance will be
(a) 20 minutes (b) 30 minutes
(c) 40 minutes (d) 25 minutes
(e) 10 minutes

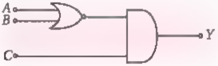
63. The ratio of the surface area of the nuclei $_{52}Te^{125}$ to that of $_{13}Al^{27}$ is
(a) $\frac{5}{3}$ (b) $\frac{125}{17}$ (c) $\frac{1}{4}$ (d) $\frac{25}{9}$
(e) $\frac{3}{5}$

64. If the frequency of incident light falling on a photosensitive metal is doubled, the kinetic energy of the emitted photoelectron is
(a) unchanged
(b) halved
(c) doubled
(d) more than twice its initial value
(e) reduced to $\frac{1}{4}$th

65. The significant result deduced from the Rutherford's scattering experiment is that
(a) whole of the positive charge is concentrated at the centre of atom
(b) there are neutrons inside the nucleus
(c) α-particles are helium nuclei
(d) electrons are embedded in the atom
(e) electrons are revolving around the nucleus

66. On an average, the number of neutrons and the energy of a neutron released per fission of a uranium atom are respectively
(a) 2.5 and 2 keV (b) 3 and 1 keV
(c) 2.5 and 2 MeV (d) 2 and 2 keV
(e) 1 and 2 MeV

67. The inputs A, B and C to be given in order to get an output $Y = 1$ from the following circuit are



(a) 0, 1, 0 (b) 1, 0, 0
(c) 1, 0, 1 (d) 1, 1, 0
(e) 0, 0, 1

68. The collector resistance and the input resistance of a *CE* amplifier are respectively 10 kΩ and 2 kΩ. If β of the transistor is 49, the voltage gain of the amplifier is
(a) 125 (b) 150 (c) 175 (d) 200
(e) 245

69. The light emitting diode (LED) is
(a) a heavily doped *p-n* junction with no external bias
(b) a heavily doped *p-n* junction with reverse bias
(c) a heavily doped *p-n* junction with forward bias
(d) a lightly doped *p-n* junction with no external bias
(e) a lightly doped *p-n* junction with reverse bias

70. A point-to-point communication mode is seen in
(a) satellite cable communication
(b) television transmission
(c) FM radio transmission
(d) AM radio transmission
(e) fax transmission

71. If the heights of transmitting and the receiving antennas are each equal to h, the maximum line-of-sight distance between them is
(R is the radius of earth)
(a) $\sqrt{2Rh}$ (b) $\sqrt{4Rh}$
(c) $\sqrt{6Rh}$ (d) $\sqrt{8Rh}$
(e) $\sqrt{Rh}$

72. The ionospheric layer acts as a reflector for the frequency range
(a) 1 kHz to 10 kHz (b) 3 to 30 MHz
(c) 3 to 30 kHz (d) 100 kHz to 1 MHz
(e) 3 GHz to 30 GHz

SOLUTIONS

1. **(d)** : Time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum.

$$\therefore \quad g = \frac{4\pi^2 L}{T^2}$$

The maximum percentage error in g is

$$\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 + 2\left(\frac{\Delta T}{T}\times 100\right)$$

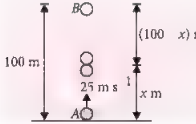
$$= 2\% + 2(3\%) = 8\%$$

2. **(c)**: Least count of a screw gauge

$$= \frac{\text{Pitch}}{\text{Number of circular scale divisions}}$$

$$= \frac{1\text{ mm}}{50} = 0.02\text{ mm}$$

3. **(e)** : Let the two balls meet at a height x m from the ground after time t s from the start.



From $S = ut + \frac{1}{2}at^2$

For ball A,

$S = x$ m, $u = 25$ m s^{-1}, $a = -g$

$\therefore\ x = 25t - \frac{1}{2}gt^2$ ... (i)

For ball B,

$S = (100 - x)$ m, $u = 0$, $a = g$

$\therefore\ 100 - x = 0 + \frac{1}{2}gt^2$ ... (ii)

Adding eqns. (i) and (ii), we get

$100 = 25t$ or $t = 4$ s

From eqn. (i),

$$x = 25\times 4 - \frac{1}{2}\times 9.8\times (4)^2 = 21.6 \text{ m}$$

$\therefore$ Distance from the top of the tower

$= (100 - x)$ m $= (100 - 21.6)$ m $= 78.4$ m

4. (c): Distance traversed in n^{th} second is

$$D_n = u + \frac{1}{2}a(2n-1)$$

Here, $u = 0$, $a = g$

$$\therefore\ D_n = \frac{1}{2}g(2n-1)$$

Distance traversed in 1st second is

$$D_1 = \frac{1}{2}g(2\times 1 - 1) = \frac{1}{2}g$$

Distance traversed in 2nd second is

$$D_2 = \frac{1}{2}g(2\times 2 - 1) = \frac{3}{2}g$$

Distance traversed in 3rd second is

$$D_3 = \frac{1}{2}g(2\times 3 - 1) = \frac{5}{2}g$$

$$\therefore\ D_1 : D_2 : D_3 = \frac{1}{2}g : \frac{3}{2}g : \frac{5}{2}g = 1:3:5$$

5. (c) : Let u be the initial velocity and a be uniform acceleration of the particle. Then

$$12 = u\times 3 + \frac{1}{2}\times a\times 3^2$$

or $24 = 6u + 9a$ ... (i)

and $42 = u\times 6 + \frac{1}{2}\times a\times 6^2$

or $42 = 6u + 18a$ ... (ii)

On solving, we get $u = 1$ m s^{-1}

6. (a) : Here, mass of the ball, $m = 10$ g $= 0.01$ kg

Let v be the velocity of the ball.

$\therefore$ Change in momentum $= mv - (-mv) = 2mv$

Impulse = Change in momentum $= 2mv$

$$\therefore\ v = \frac{\text{Impulse}}{2m} = \frac{0.54 \text{ N s}}{2\times 0.01 \text{ kg}} = 27 \text{ m s}^{-1}$$

7. (b) : Here, $\vec{r} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{p} = 2\hat{i} - \hat{j} + \hat{k}$

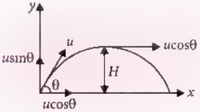
Angular momentum, $\vec{L} = \vec{r}\times\vec{p}$

$$\therefore\ \vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-2+1) + \hat{j}(2-1) + \hat{k}(-1+4)$$

$$= -\hat{i} + \hat{j} + 3\hat{k}$$

8. (d) :



At maximum height (H) the vertical component of the projectile becomes zero whereas its horizontal component remains the same. *i.e.* $u\cos\theta$

9. (a) : Here, $x = 4t^2$, $y = 3t^2$

$$\therefore\ v_x = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$$

and $v_y = \frac{dy}{dt} = \frac{d}{dt}(3t^2) = 6t$

The speed of the particle at any time t is

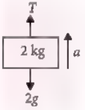
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(8t)^2 + (6t)^2} = 10t$$

10. (b) : The cyclist bends while taking turn in order to provide necessary centripetal force.

11. (d) : Let a be common acceleration of the system and T be tension in the string.

$$\therefore\ a = \frac{F - 4g - 2g}{4+2} = \frac{120 - 40 - 20}{6} = 10 \text{ m s}^{-2}$$

The free body diagram of 2 kg block is as shown in figure.



$\therefore\ T - 2g = 2a$

$T = 2(a + g) = 2(10 + 10)$

$= 40$ N

BRAIN MAP

UNITS AND MEASUREMENT

Engineering and science are based on measurements and [illegible] comparisons. One purpose of physics and engineering is to design and conduct those experiments.

Units

- Unit of a physical quantity is defined as the standard of measurement of that quantity.

(i) **Fundamental units**: The units selected for measurement of the base quantities are known as the base units or fundamental units.

(ii) **Derived units** : The physical quantities derived from the fundamental quantities are derived quantities and their units are known as derived units.

(iii) **Supplementary units** : Units of plane angle and solid angle are known as supplementary units. It is of two types.

(a) **Radian (rad)** : It is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of circle.

(b) **Steradian (sr)** : It is defined as the solid angle subtended at the centre of a sphere by a surface of the sphere which is equal in area to that of a square having each side equal to the radius of the sphere.

Errors in measurement

- **Error** : The error in a measurement is equal to the difference between the true value and the measured value of the quantity

Error = True value - Measured value

- **Types of error** : Systematic errors are those errors that tend to be in one direction, either positive or negative. Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible. Random errors are the errors, which occur irregularly. They are random with respect to sign and size. Least count error : The smallest value that can be measured by the measuring instrument is called its least count and error associated with resolution of the instrument is known as least count error.

Dimension of physical quantity

- The dimensions of a physical quantity are the powers (or exponents) to which the fundamental quantities must be raised to represent that quantity completely.

Dimensional formulae and dimensional equation

- **Dimensional formula** : The expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity is called the dimensional formulae of the physical quantity.
- **Dimensional equation** : The equation obtained by equating a physical quantity with its dimensional formulae is called the dimensional equation of the given physical quantity.

Applications of dimensional analysis

- The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis. Following are the three main uses of dimensional analysis.

(i) To convert a physical quantity from one system of units to another.

(ii) To check the correctness of a given physical relation.

(iii) To derive a relationship between different physical quantities.

- **Homogeneity principle** : According to this principle a physical equation will be dimensionally correct if the dimension of all the terms occurring on both sides of the equation are the same.
- **Conversion of one system of units to another** : It is based on the fact that the magnitude of a physical quantity remain the same whatever may be the system of units.

Measurement of length

- **Length** : It is the measure of the distance between two points in space.
- Direct methods for the measurement of length : a metre scale is used for measuring lengths from 10^{-3} m to 10^{2} m, a vernier callipers is used for measuring lengths upto 10^{-4} m and a screw gauge and spherometer are used to measure lengths upto 10^{-5} m.
- Indirect methods for large distances : echo (reflection) method, triangulation method, parallax method.
- Indirect method for small distances : **avogadro's hypothesis** and **tunnelling microscopy**.

Measurement of mass

- **Inertial mass** : It is a measure of the reluctance *i.e.*, inertia on the part of the object to change its motion when a force is applied on it.
- **Gravitational mass** : It is a measure of the pull on the object exerted by the earth.

Measurement of time

- According to Einstein, "Time is simply what a clock reads". Any phenomenon that repeats itself after equal intervals of time can be used as a time standard.

Absolute error, Relative error and Percentage error

- **Absolute error** : The magnitude of the difference between the true value of the quantity measured and the individual measured value is called absolute error.
- **Mean absolute error** : It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by Δa_{mean}.

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}, \quad \Delta a_{mean} = \frac{1}{n} \times \sum_{i=1}^{n} |\Delta a_i|$$

- Mean value of measurement $a_{mean} = \frac{a_1 + a_2 + ... + a_n}{n}$, $a_{mean} = \frac{1}{n}\sum_{i=1}^{n} a_i$

The final value of measurement may be written as $a = a_{mean} \pm \Delta a_{mean}$

- **Relative error or fractional error** $= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\Delta a_{mean}}{a_{mean}}$
- **Percentage error** : When the relative error is expressed in percentage it is known as percentage error

Percentage error, $\delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$

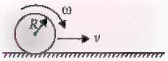
Significant figures

- The number of digits in a measurement about which one reasonably sure plus the first uncertain digit are called significant digits or significant figures.
- **Rules to find the significant figures** **Rule-1.** All non zero digits are significant. *e.g.* 1324 has four significant figures. **Rule-2** : All zeros occurring between two non zero digits are significant. *e.g.* 120024 has 6 significant digits. **Rule-3** : If the number is less than 1, the zero(s) on the right of decimal point, but to the left of the first non-zero digit are not significant. *e.g.* 0.00064 has two significant digits. **Rule-4** : In a number without a decimal point the terminal or trailing zero(s) are not significant *e.g.* 227800 has four significant digits. **Rule-5** : In a number with a decimal point the trailing zero(s) are significant. *e.g.* 3.200 or 0.05400 have four significant digits each. **Note** : The power (or exponent) of 10 is irrelevant to the determination of significant figures. For example, 3.100×10^{2} has 4 significant figures. The change of units only changes the order of exponent but not the number of significant figures. *e.g.* 1.40 m $= 1.40 \times 10^{2}$ cm both have three significant figures.
- **Rounding off - Rule-1**: If the digit to be dropped is less than 5, then the preceding digit is left unchanged, *e.g.* 8.22 is rounded off to 8.2. **Rule-2**: If the digit to be dropped is more than 5, then the preceding digit is raised by one. *e.g.* 6.87 is rounded off to 6.9. **Rule-3**: If the digit to be dropped is 5 followed by digit other than zero, then the preceeding digit is raised by one. *e.g.* 7.851 is rounded off to 7.9. **Rule-4**: If the digit to be dropped is 5 or 5 followed by zero, then preceding digit is left unchanged, if it is even. *e.g.* 5.250 rounding off to 5.2. **Rule-5**: If the digit to be dropped is 5 or 5 followed by zero, then the preceding digit is raised by one, if it is odd. *e.g.* 3.750 is rounded off to 3.8.

Rules for arithmetic operations with significant figures

- In both, addition or subtraction the final result should retain as many decimal places as are there in the number with least decimal places. *e.g.* 24.36 + 0.0623 + 256.2 = 280.6223. The result should be rounded off to 280.6
- In multiplication or division, the final result should retain as many significant figures as there in the original number with the least significant figures. *e.g.* 4.6 × 0.128 = 0.5888. The result should be rounded off to 0.59

12. (a) :



Here, mass of the sphere, $M = 300$ g $= 0.3$ kg
In case of rolling motion without slipping
Total energy, $K = K_T + K_R$

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}MR^2\omega^2$$

$$\left(\because \text{For solid sphere, } I = \frac{2}{5}MR^2\right)$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 \qquad (\because v = R\omega)$$

$$= \frac{7}{10}Mv^2 = \frac{7}{10}\times 0.3\times 5^2 = 5.25 \text{ J}$$

13. (a) : Let the bullet be fired with velocity v.
For first part of penetration,
Using 3rd equation of motion, we get

$$\left(\frac{v}{2}\right)^2 - (v)^2 = 2a(20)$$

$$-\frac{3}{4}v^2 = 40a \quad \text{or} \quad a = -\frac{3v^2}{160} \qquad \text{... (i)}$$

For latter part of penetration,
Let x be distance travelled by the bullet before it comes to rest.
Again, using 3rd equation of motion, we get

$$0 - \left(\frac{v}{2}\right)^2 = 2ax$$

$$x = -\frac{v^2}{8a} = -\frac{v^2}{8}\left(\frac{160}{-3v^2}\right) = 6.66 \text{ cm} \quad \text{(Using (i))}$$

14. (a) : In elastic collision both momentum and kinetic energy are conserved.

15. (e) : According to law of conservation of angular momentum, we get

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

Substituting the given values, we get

$$I_1 \times 2 + 1\times 10^{-3} \times 5 = (I_1 + 1\times 10^{-3}) \times 4$$

$$2I_1 + 5\times 10^{-3} = 4I_1 + 4\times 10^{-3}$$

$$2I_1 = 1\times 10^{-3}$$

$$I_1 = \frac{1\times 10^{-3}}{2} = 0.5\times 10^{-3} \text{ kg m}^2$$

16. (c): Here,

Initial angular speed, $\omega_0 = \dfrac{2\pi\times 1800}{60}$ rad s^{-1}

$= 60\pi$ rad s^{-1}

Final angular speed,
$\omega = 0$ (As wheel comes to rest)
Time, $t = 2$ minutes $= 120$ s

$\therefore$ Angular retardation $= \dfrac{\omega_0 - \omega}{t}$

$$= \frac{60\pi - 0}{120} = \frac{\pi}{2} \text{ rad s}^{-2}$$

Note : In the question paper the unit of angular retardation is given as rad s^{-1}. But it is wrong. The correct unit of angular retardation is rad s^{-2}.

17. (a) : Let the particle of mass m moves with uniform speed v along the circular path of radius r.

$\therefore$ Angular momentum of the particle, $L = mvr$

$$\text{or} \quad v = \frac{L}{mr} \qquad \text{... (i)}$$

Centripetal force acting on the particle is

$$F = \frac{mv^2}{r} = \frac{m\left(\frac{L}{mr}\right)^2}{r} \qquad \text{(Using(i))}$$

$$= \frac{L^2}{mr^3}$$

Note : You can also check it dimensionally, only option (a) has the dimensions of force.

18. (e) : Gravitational potential energy
= gravitation potential × mass of the body
Except (e), all other statements are correct.

19. (c): Gravitational potential energy of the body of mass m on the surface of earth is

$$U_s = -\frac{GMm}{R}$$

where M is the mass of the earth.
Gravitational potential energy of the body at a height h ($= R$) from the surface of earth is

$$U_h = -\frac{GMm}{R+h} = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$\therefore$ Required amount of energy $= U_h - U_s$

$$= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}\left(1-\frac{1}{2}\right)$$

$$= \frac{GMm}{2R} = \frac{mgR}{2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

20. (c) : Kinetic energy of the satellite $= \frac{1}{2}mv^2$

Total energy of the satellite = – Kinetic energy of the satellite

$$= -\frac{1}{2}mv^2$$

21. (a) : Let V_1 and V_2 be the volumes of two soap bubbles of radii r_1 and r_2 respectively. Then

$$V_1 = \frac{4}{3}\pi r_1^3 \quad \text{and} \quad V_2 = \frac{4}{3}\pi r_2^3$$

Let S be the surface tension of the soap solution. If P_1 and P_2 are the excess pressure inside the two soap bubbles, then

$$P_1 = \frac{4S}{r_1} \quad \text{and} \quad P_2 = \frac{4S}{r_2}$$

When these two bubbles coalesce under isothermal conditions a bigger bubble of radius R is formed. If V and P be the volume and excess pressure inside this bigger bubble, then

$$V = \frac{4}{3}\pi R^3 \quad \text{and} \quad P = \frac{4S}{R}$$

As the bigger bubble is formed under isothermal conditions, so Boyle's law holds.

i.e., $P_1V_1 + P_2V_2 = PV$

$$\frac{4S}{r_1} \times \frac{4}{3}\pi r_1^3 + \frac{4S}{r_2} \times \frac{4}{3}\pi r_2^3 = \frac{4S}{R} \times \frac{4}{3}\pi R^3$$

$$r_1^2 + r_2^2 = R^2 \quad \text{or} \quad R = \sqrt{r_1^2 + r_2^2}$$

22. (b) : The ratio of hydraulic stress to the corresponding strain is known as bulk modulus.

23. (a) : Pressure difference between lungs and atmosphere

= (760 – 750) mm of Hg

= 10 mm of Hg = 1 cm of Hg

Let the boy can suck water from depth h. Then

Pressure difference = $h\rho_{water}g$ = 1 cm of Hg

$h \times 1\ \text{g cm}^{-3} \times 980\ \text{cm s}^{-2}$

$= 1\ \text{cm} \times 13.6\ \text{g cm}^{-3} \times 980\ \text{cm s}^{-2}$

$\therefore\ h = 13.6$ cm

24. (c): Potential energy stored in a spring is

$$U = \frac{1}{2}kx^2$$

where x is the extension (or compression) in the spring.

Let k N m^{-1} be spring constant of the spring.

As per question,

$$U = \frac{1}{2}k(1\times10^{-3}\ \text{m})^2 = 1\ \text{J} \qquad \text{... (i)}$$

It is further compressed by 1 mm. Then

$$U' = \frac{1}{2}k(2\times10^{-3}\ \text{m})^2 \qquad \text{... (ii)}$$

Divide eqn. (ii) by eqn. (i), we get

$$\frac{U'}{U} = 4 \quad \text{or} \quad U' = 4U$$

$\therefore$ Work done, $W = U' - U = 4U - U$

$= 3U = 3 \times 1\ \text{J} = 3\ \text{J}$

25. (d) : According to kinetic energy of gases

Root mean square velocity (rms) of gas molecules is

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

where m is the mass of each molecule, T is the absolute temperature and k_B is the Boltzmann constant.

$$\therefore\ v_{rms} \propto \sqrt{\frac{T}{m}} \quad \text{or} \quad v_{rms} \propto m^{-1/2}\,T^{1/2}$$

26. (b) : Efficiency of a Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\therefore\ 0.2 = 1 - \frac{T_2}{T_1} \quad \text{or} \quad \frac{T_2}{T_1} = 0.8 \qquad \text{... (i)}$$

When T_2 is reduced by 50 K, its efficiency becomes 0.4.

$$\therefore\ 0.4 = 1 - \frac{T_2 - 50}{T_1} \quad \text{or} \quad \frac{T_2 - 50}{T_1} = 0.6 \qquad \text{... (ii)}$$

Divide eqn. (i) by eqn. (ii), we get

$$\frac{T_2}{T_2 - 50} = \frac{0.8}{0.6} = \frac{4}{3}$$

$3T_2 = 4T_2 - 200$ or $T_2 = 200$ K

From eqn. (i), $T_1 = \frac{T_2}{0.8} = \frac{200\text{ K}}{0.8} = 250$ K

27. (c) : Here, number of degrees of freedom, $f = 6$
Molar specific heat of the gas at constant volume,

$$C_V = \frac{f}{2}R = \frac{6}{2}R = 3R$$

28. (c): A rigid diatomic molecule has 5 degrees of freedom : 3 translational and 2 rotational.

29. (a) : The given equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} + 2y = 0 \quad \text{or} \quad \frac{d^2y}{dt^2} = -2y$$

Compare it with standard equation of simple harmonic motion,

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

we get, $\omega^2 = 2$ or $\omega = \sqrt{2}$

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}$ s

30. (b) : The effective spring constant K of springs in parallel is
$K = K_1 + K_2 + ...$
Except (b), all other statements are correct.

31. (b) : In SHM,

Total energy, $E = \frac{1}{2}m\omega^2A^2$

Kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

where x is the distance from the mean position.
At $x = 0.707A$

$$K = \frac{1}{2}m\omega^2(A^2 - (0.707A)^2) = \frac{1}{2}m\omega^2(0.5A^2)$$

As per question, $E = 100$ J

$$\therefore \; K = 0.5\left(\frac{1}{2}m\omega^2A^2\right) = 0.5 \times 100 \text{ J} = 50 \text{ J}$$

32. (e) : Here,
$y_1 = A\sin[k(x + ct)] = A\sin(kx + kct)$
$y_2 = A\sin[k(x - ct)] = A\sin(kx - kct)$

According to principle of superposition, the resultant wave on the string is

$y = y_1 + y_2$
$= A\sin(kx + kct) + A\sin(kx - kct)$
$= A[\sin(kx + kct) + \sin(kx - kct)]$
$= 2A\sin kx \cos kct$
$[\because \sin(C + D) + \sin(C - D) = 2\sin C \cos D]$

The amplitude of this wave is
$A_R = 2A\sin kx$

The positions of antinodes (where amplitude is maximum) are given by
$|\sin kx| = 1$

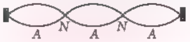
$$\therefore \; kx = \left(n + \frac{1}{2}\right)\pi, \text{ where } n = 0, 1, 2, ...$$

$$\text{or} \quad x = \left(n + \frac{1}{2}\right)\frac{\pi}{k}$$

$$x = \frac{\pi}{2k}, \frac{3\pi}{2k}, \frac{5\pi}{2k}, ...$$

$\therefore$ The distance between adjacent antinodes is π/k.

33. (d) : The vibration of a stretched wire in the third harmonics or second overtone is as shown in the figure.



From the figure,
Number of nodes = 2
Number of antinodes = 3

34. (d) : In a tube open at both ends, all the harmonics are present.
In a tube open at one end, only odd harmonics are present.
The distance between successive nodes is equal to half the wavelength.
Reflection of a wave from a rigid wall changes the phase by 180° or π.
Except (d), all the other statements are wrong.

35. (c): As the electron experiences the force directed away from the sheet therefore the sheet must be negatively charged.
Let σ be uniform surface density of the sheet.

Electric field due to the sheet is $E = \frac{\sigma}{2\varepsilon_0}$

Force experienced by the electron is

$$F = eE = \frac{e\sigma}{2\varepsilon_0} \quad \text{or} \quad \sigma = \frac{2F\varepsilon_0}{e}$$

$$\sigma = \frac{2\times1.6\times10^{-12}\times8.854\times10^{-12}}{1.6\times10^{-19}}$$

$= 17.708 \times 10^{-5}\ \mathrm{C\,m^{-2}} = 177.08 \times 10^{-6}\ \mathrm{C\,m^{-2}}$

$= 177.08\ \mu\mathrm{C\,m^{-2}}$

Area of the sheet, $A = (0.5\ \mathrm{m})^2 = 0.25\ \mathrm{m}^2$

Total charge on the sheet is

$Q = \sigma A = (177.08\ \mu\mathrm{C\,m^{-2}})\,(0.25\ \mathrm{m^2})$

$= 44.27\ \mu\mathrm{C}$

It is negative.

$\therefore\ Q = -\,44.27\ \mu\mathrm{C}$

36. (d) : The energy U stored in a capacitor of capacitance C having a charge Q under a potential V is

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} \qquad (\text{As } Q = CV)$$

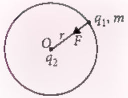
37. (e) : According to Coulomb's law the electrostatic force between two point charges q_1 and q_2 separated by a distance r in a medium is

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}$$

where ε is the permittivity of the medium.

$F \propto q_1q_2$

38. (d) :



According to Coulomb's law the force of attraction between these two charges is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

This force provides the required centripetal force for circular motion.

$$\therefore\ \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\text{or}\quad v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{mr}} \qquad \text{...(i)}$$

Time period of revolution

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{mr}}} \qquad \text{(Using (i))}$$

$$= 2\pi r\sqrt{\frac{4\pi\varepsilon_0 mr}{q_1q_2}} = \sqrt{\frac{16\pi^3\varepsilon_0 mr^3}{q_1q_2}}$$

39. (c): As $F = qE$ or $E = \frac{F}{q}$

Here, $F = 1000$ N, $q = 2$ C

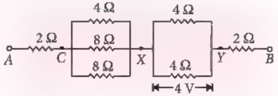
$$\therefore\ E = \frac{1000\ \mathrm{N}}{2\ \mathrm{C}} = 500\ \mathrm{N\,C^{-1}}$$

As $E = \frac{V}{d}$ or $V = Ed$

Here, $d = 2\ \mathrm{cm} = 2 \times 10^{-2}$ m

$\therefore\ V = (500\ \mathrm{N\,C^{-1}})\,(2 \times 10^{-2}\ \mathrm{m}) = 10$ V

40. (a) : The equivalent circuit of given network is as shown in the figure.



Resistors 4 Ω, 8 Ω and 8 Ω are connected in parallel.

$\therefore$ Resistance between C and X is

$$\frac{1}{R_{CX}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{2+1+1}{8} = \frac{4}{8} = \frac{1}{2}$$

or $R_{CX} = 2\ \Omega$

Resistors 4 Ω and 4 Ω are connected in parallel.

$\therefore$ Resistance between X and Y is

$$\frac{1}{R_{XY}} = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

or $R_{XY} = 2\ \Omega$

Total resistance between A and B is

$R_{AB} = R_{AC} + R_{CX} + R_{XY} + R_{YB}$

$= 2\ \Omega + 2\ \Omega + 2\ \Omega + 2\ \Omega = 8\ \Omega$

Current through X to $Y = \frac{4\ \mathrm{V}}{2\ \Omega} = 2$ A

$\therefore$ Potential difference across AB

$= (2\ \mathrm{A})(8\ \Omega) = 16$ V

41. (b) : The ammeter will show a zero reading, if the potential difference across $R = 2$ V

$$\therefore \quad \frac{10}{500+R} \times R = 2$$

$$10R = 1000 + 2R$$

or $8R = 1000$ or $R = 125\ \Omega$

Note : In the question paper emf of battery is given as 10 Ω instead of 10 V.

42. (d) : Here, emf of each cell = E

Internal resistance of each cell = r

Number of cells, $n = 5$

In series,

The current through the external resistance R is

$$I = \frac{nE}{nr+R} = \frac{5E}{5r+R} \quad \text{... (i)}$$

In parallel,

The current through the external resistance R is

$$I' = \frac{E}{\frac{r}{n}+R} = \frac{nE}{r+nR} = \frac{5E}{r+5R} \quad \text{... (ii)}$$

As per question, $I = I'$

$$\therefore \quad \frac{5E}{5r+R} = \frac{5E}{r+5R} \quad \text{or} \quad 5r+R = r+5R$$

or $R = r$ or $\frac{R}{r} = 1$

43. (b) : Consider a device R to which a power P is delivered via transmission cables having a distance R_c. If V is the voltage across R and I the current through it, then

$$P = VI \quad \text{... (i)}$$

The power dissipated in the transmission cables is

$$P_c = I^2 R_c = \frac{P^2 R_c}{V^2} \quad \text{(Using (i))}$$

Thus, the power dissipated in the transmission cables is inversely proportional to V^2.

44. (a) : Change in internal energy of the gas is equal to the heat produced due to current flowing.

$$\therefore \quad \Delta U = I^2 Rt = (1\text{ A})^2 (50\ \Omega)(2 \times 60\text{ s})$$

$$= 6 \times 10^3\text{ J} = 6\text{ kJ}$$

45. (d)

46. (e) : In a vibration magnetometer

Time period, $T = 2\pi\sqrt{\frac{I}{MB}}$

where the symbols have their usual meanings.

or $M = \frac{4\pi^2 I}{BT^2}$

Here, $I = 9 \times 10^{-5}$ kg m^2, $B = 16\pi^2 \times 10^{-5}$ T

$$T = \frac{15}{20} = \frac{3}{4}\text{ s}$$

$$\therefore \quad M = \frac{4\pi^2 \times 9 \times 10^{-5}}{16\pi^2 \times 10^{-5} \times \left(\frac{3}{4}\right)^2} = 4\text{ A m}^2$$

47. (d)

48. (c): Kinetic energy of the proton, $K = \frac{q^2 B^2 r^2}{2m}$

As $\upsilon = \frac{qB}{2\pi m}$ or $qB = 2\pi m\upsilon$

$$\therefore \quad K = \frac{(2\pi m\upsilon)^2 r^2}{2m} = 2\pi^2 m\upsilon^2 r^2$$

49. (b) : Work done in rotating the magnetic dipole of moment M in a magnetic field B from position θ_1 to θ_2 is

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°$, $\theta_2 = 180°$

$$\therefore \quad W = MB(\cos 0° - \cos 180°) = 2MB$$

50. (c): Lenz's law gives the polarity of induced emf.

51. (a) : In an LCR series circuit, the phase difference ϕ between the current and voltage is

$$\tan\phi = \frac{X_C - X_L}{R}$$

At resonance, $X_C = X_L$

$\therefore \quad \phi = 0°$

Hence, the current and voltage are in phase.

52. (b) : The maximum induced emf is

$$\varepsilon_{max} = BA\omega$$

Here, $B = 0.01$ T, $A = \pi R^2 = \pi \times (1\text{ m})^2 = \pi\text{ m}^2$

$\omega = 100$ rad s^{-1}

$$\therefore \quad \varepsilon_{max} = 0.01 \times \pi \times 100\text{ V} = \pi\text{ V}$$

53. **(e)** : As $\frac{I_s}{I_p}=\frac{N_p}{N_s}$ $\quad\therefore\quad N_s=\frac{I_p}{I_s}N_p$

where the subscripts p and s refer to primary and secondary coils respectively.
Here, $I_p = 4$ A, $I_s = 24$ A, $N_p = 330$

$$\therefore\quad N_s=\frac{4}{24}\times 330=55$$

54. **(d)** : The average energy density due to magnetic field is

$$u_B=\frac{1}{2}\frac{B_{rms}^2}{\mu_0}=\frac{1}{2\mu_0}\left(\frac{B_0}{\sqrt{2}}\right)^2 \qquad \left(\because B_{rms}=\frac{B_0}{\sqrt{2}}\right)$$

$$=\frac{1}{4}\frac{B_0^2}{\mu_0}=\frac{1}{4}\frac{(E_0/c)^2}{\mu_0} \qquad \left(\because B_0=\frac{E_0}{c}\right)$$

$$=\frac{1}{4}\frac{E_0^2}{c^2\mu_0}=\frac{1}{4\mu_0}\frac{E_0^2}{(1/\mu_0\varepsilon_0)} \qquad \left(\because c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

$$=\frac{1}{4}\varepsilon_0E_0^2=\frac{1}{4}\times 8.854\times 10^{-12}\times(1)^2$$

$$=2.21\times 10^{-12}\ \text{J m}^{-3}$$

Note : In the question paper the unit of average energy density is given as J m^{-2}. But it is wrong. The correct unit of average energy density is J m^{-3}.

55. **(b)** : The electric and magnetic fields are in the same phase.
The energy is equally divided between electric and magnetic fields.
The direction of propagation is perpendicular to both electric and magnetic fields.
The pressure exerted by the wave is equal to the energy density of the wave.
The speed of the wave is E/B.

56. **(a)** : The apparent flattening (oval shape) of the sun at sunset and sunrise is due to refraction.

57. **(c)**: According to Brewster's law, $\mu = \tan i_p$
where i_p is the polarising angle of the medium.
Here, $i_p = 60°$

$$\therefore\quad \mu=\tan 60°=\sqrt{3} \qquad \text{... (i)}$$

Let C be the critical angle of the medium.

$$\therefore\quad \sin C=\frac{1}{\mu}=\frac{1}{\sqrt{3}} \qquad \text{(Using (i))}$$

$$\text{or}\quad C=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

58. **(b)** : For rear view- Convex mirror.

59. **(a)** : Fringe width, $\beta=\frac{\lambda D}{d}$

where, λ = Wavelength of the source
D = Distance between screen and the slits
d = Distance between the two slits

60. **(b)** : Angular position of n^{th} minimum from the central maximum is

$$\sin\theta_n=\frac{n\lambda}{a}$$

Here, $\lambda = 5000$ Å $= 5\times 10^{-7}$ m
$a = 2.5\times 10^{-4}$ cm $= 2.5\times 10^{-6}$ m, $n = 2$

$$\therefore\quad \sin\theta_2=\frac{2\times 5\times 10^{-7}\ \text{m}}{2.5\times 10^{-6}\ \text{m}}=\frac{2}{5}$$

$$\text{or}\quad \theta_2=\sin^{-1}\left(\frac{2}{5}\right)$$

61. **(e)** : de Broglie wavelength, $\lambda=\frac{h}{\sqrt{2mqV}}$

Since potential (V) is same for both the particles.

$$\therefore\quad \lambda\propto\frac{1}{\sqrt{mq}}$$

$$\text{Thus,}\quad \frac{\lambda_e}{\lambda_p}=\sqrt{\frac{m_pq_p}{m_eq_e}}=\sqrt{\frac{m_p}{m_e}} \qquad (\because q_p=q_e=e)$$

62. **(c)**: Here, $T_{1/2} = 20$ minutes
Let t_1 be the time taken when 50% of the substance decay.
$\therefore$ Amount of substance left undecayed $= 50\% = \frac{1}{2}$

$$\text{As}\quad \frac{N}{N_0}=\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$\therefore\quad \frac{1}{2}=\left(\frac{1}{2}\right)^{t_1/T_{1/2}} \qquad \text{... (i)}$$

Let t_2 be time taken when 87.5% of the substance decay.

$\therefore$ Amount of substance left undecayed

$$= 12.5\% = \frac{1}{8}$$

$$\therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^{t_2/T_{1/2}} \qquad \text{...(ii)}$$

Divide eqn. (ii) by eqn. (i), we get

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{(t_2-t_1)/T_{1/2}} \text{ or } \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{(t_2-t_1)/T_{1/2}}$$

$$\therefore \quad \frac{t_2 - t_1}{T_{1/2}} = 2$$

or $t_2 - t_1 = 2T_{1/2} = 2 \times 20$ minutes

$= 40$ minutes

63. **(d)** : Nucleus radius, $R = R_0 A^{1/3}$
where R_0 is a constant and A is the mass number

$$\therefore \quad \frac{R_{\text{Te}}}{R_{\text{Al}}} = \left(\frac{A_{\text{Te}}}{A_{\text{Al}}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3}$$

Assuming nucleus to be spherical

$\therefore$ Surface area, $S = 4\pi R^2$

$$\therefore \quad \frac{S_{\text{Te}}}{S_{\text{Al}}} = \left(\frac{R_{\text{Te}}}{R_{\text{Al}}}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

64. **(d)** : According to Einstein's photoelectric equation
The kinetic energy of the emitted photoelectron is

$$K = h\upsilon - \phi_0 \qquad \text{... (i)}$$

where υ is the frequency of incident radiation and ϕ_0 is a work function of the metal.
If the frequency of incident radiation is doubled, then

$$K' = 2h\upsilon - \phi_0 = 2(h\upsilon - \phi_0) + \phi_0$$

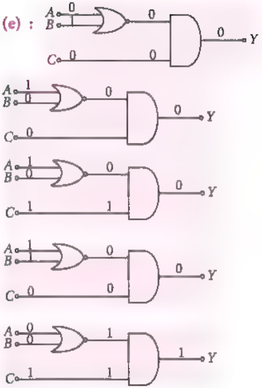
$$= 2K + \phi_0 \qquad \text{(Using (i))}$$

$K' > 2K$

65. **(a)** : The significant result deduced from the Rutherford's scattering is that whole of the positive charge is concentrated at the centre of atom.

66. **(c)**: On an average 2.5 neutrons are released per fission of the uranium atom.
The energy of the neutron released per fission of the uranium atom is 2 MeV.

67. **(e)** :



68. **(e)** : Here, output resistance, $R_0 = 10\ \text{k}\Omega$
Input resistance, $R_i = 2\ \text{k}\Omega$

$$\beta = 49$$

$$\text{Voltage gain, } A_V = \beta \times \frac{R_0}{R_i} = 49 \times \frac{10}{2} = 245$$

69. **(c)**: The light emitting diode is a heavily doped p-n junction which under forward bias emits spontaneous radiation.

70. **(e)** : In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver.

71. **(d)** : The maximum line-of-sight distance between the transmitting and receiving antennas is

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_T and h_R are the heights of transmitting and receiving antennas respectively.
Here, $h_T = h_R = h$

$$\therefore d_M = \sqrt{2Rh} + \sqrt{2Rh} = 2\sqrt{2Rh} = \sqrt{8Rh}$$

72. **(b)** : The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape.

SOLVED PAPER 2014

West Bengal JEE

CATEGORY-I

Q.1 to Q.45 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark.

1. A small metal sphere of radius a is falling with a velocity v through a vertical column of a viscous liquid. If the coefficient of viscosity of the liquid is η, then the sphere encounters an opposing force of

(a) $6\pi\eta a^2 v$ (b) $\frac{6\eta v}{\pi a}$

(c) $6\pi\eta a v$ (d) $\frac{\pi\eta v}{6a^3}$

2. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(a) $\frac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1}$ (b) $\frac{h_1 t_1^2 + h_2 t_2^2}{h_2 t_1 + h_1 t_2}$

(c) $\frac{h_1 t_2^2 + h_2 t_1^2}{h_1 t_2 + h_2 t_1}$ (d) $\frac{h_1 t_1^2 - h_2 t_2^2}{h_1 t_1 - h_2 t_2}$

3. In which of the following pairs, the two physical quantities have different dimensions?

(a) Planck's constant and angular momentum

(b) Impulse and linear momentum

(c) Moment of inertia and moment of a force

(d) Energy and torque

4. A very small circular loop of radius a is initially (at $t = 0$) coplanar and concentric with a much larger fixed circular loop of radius b. A constant current I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is

(a) $\frac{\pi a^2 \mu_0 I}{2b}\omega\cos(\omega t)$

(b) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin(\omega^2 t^2)$

(c) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin(\omega t)$

(d) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin^2(\omega t)$

5. An electron in a circular orbit of radius 0.05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in A m^2)

(a) 2.16×10^{-23} (b) 3.21×10^{-22}

(c) 3.21×10^{-24} (d) 1.26×10^{-23}

6. A drop of some liquid of volume 0.04 cm^3 is placed on the surface of a glass slide. Then another glass slide is placed on it in such a way that the liquid forms a thin layer of area 20 cm^2 between the surfaces of the two slides. To separate the slides a force of 16×10^5 dyne has to be applied normal to the surfaces. The surface tension of the liquid is (in dyne cm^{-1})

(a) 60 (b) 70 (c) 80 (d) 90

7. In a transistor output characteristics commonly used in common emitter configuration, the base current I_B, the collector current I_C, and the collector-emitter voltage V_{CE} have values of

the following orders of magnitude in the active region

(a) I_B and I_C both are in μA, and V_{CE} in volts
(b) I_B is in μA and I_C is in mA and V_{CE} in volts
(c) I_B is in mA and I_C is in μA and V_{CE} in mV
(d) I_B is in mA and I_C is in mA and V_{CE} in mV

8. If n denotes a positive integer, h the Planck's constant, q the charge and B the magnetic field, then the quantity $\left(\dfrac{nh}{2\pi qB}\right)$ has the dimension of

(a) area (b) length
(c) speed (d) acceleration

9. In the circuit shown assume the diode to be ideal. When V_i increases from 2 V to 6 V, the change in the current is (in mA)

(a) zero (b) 20 (c) 80/3 (d) 40

10. A galvanometer having internal resistance 10 Ω requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full-scale deflection at 120 V, we need to connect a resistance of

(a) 11990 Ω in series (b) 11990 Ω in parallel
(c) 12010 Ω in series (d) 12010 Ω in parallel

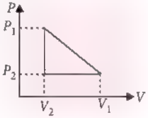
11. Three capacitors 3 μF, 6 μF and 6 μF are connected in series to a source of 120 V. The potential difference, in volts, across the 3 μF capacitor will be

(a) 24 (b) 30 (c) 40 (d) 60

12. Consider three vectors $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{C}=2\hat{i}-3\hat{j}+4\hat{k}$. A vector $\vec{X}$ of the form $\alpha\vec{A}+\beta\vec{B}$ (α and β are numbers) is perpendicular to $\vec{C}$. The ratio of α and β is

(a) 1 : 1 (b) 2 : 1
(c) −1 : 1 (d) 3 : 1

13. One mole of a van der Waals' gas obeying the equation $\left(P+\dfrac{a}{V^2}\right)(V-b)=RT$ undergoes the quasi-static cyclic process which is shown in the P-V diagram. The net heat absorbed by the gas in this process is



(a) $\frac{1}{2}(P_1-P_2)(V_1-V_2)$

(b) $\frac{1}{2}(P_1+P_2)(V_1-V_2)$

(c) $\frac{1}{2}\left(P_1+\frac{a}{V_1^2}-P_2-\frac{a}{V_2^2}\right)(V_1-V_2)$

(d) $\frac{1}{2}\left(P_1+\frac{a}{V_1^2}+P_2+\frac{a}{V_2^2}\right)(V_1-V_2)$

14. A scientist proposes a new temperature scale in which the ice point is 25 X (X is the new unit of temperature) and the steam point is 305 X. The specific heat capacity of water in this new scale is (in J kg^{-1} X^{-1})

(a) 4.2×10^3 (b) 3.0×10^3
(c) 1.2×10^3 (d) 1.5×10^3

15. The intensity of magnetization of a bar magnet is 5.0×10^4 A m^{-1}. The magnetic length and the area of cross section of the magnet are 12 cm and 1 cm^2 respectively. The magnitude of magnetic moment of this bar magnet is (in SI unit)

(a) 0.6 (b) 1.3 (c) 1.24 (d) 2.4

16. An infinite sheet carrying a uniform surface charge density σ lies on the xy-plane. The work done to carry a charge q from the point $\vec{A}=a(\hat{i}+2\hat{j}+3\hat{k})$ to the point $\vec{B}=a(\hat{i}-2\hat{j}+6\hat{k})$ (where a is a constant with the dimension of length and ε_0 is the permittivity of free space) is

(a) $\dfrac{3\sigma aq}{2\varepsilon_0}$ (b) $\dfrac{2\sigma aq}{\varepsilon_0}$

(c) $\dfrac{5\sigma aq}{2\varepsilon_0}$ (d) $\dfrac{3\sigma aq}{\varepsilon_0}$

17. A luminous object is separated from a screen by distance d. A convex lens is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is

(a) $4d$ (b) $2d$ (c) $d/2$ (d) $d/4$

18. Four cells, each of emf E and internal resistance r, are connected in series across an external resistance R. By mistake one of the cells is connected in reverse. Then the current in the external circuit is

(a) $\frac{2E}{4r+R}$ (b) $\frac{3E}{4r+R}$

(c) $\frac{3E}{3r+R}$ (d) $\frac{2E}{3r+R}$

19. Consider two concentric spherical metal shells of radii r_1 and r_2 ($r_2 > r_1$). If the outer shell has a charge q and the inner one is grounded, the charge on the inner shell is

(a) $\frac{-r_2}{r_1}q$ (b) zero

(c) $\frac{-r_1}{r_2}q$ (d) $-q$

20. Consider a blackbody radiation in a cubical box at absolute temperature T. If the length of each side of the box is doubled and the temperature of the walls of the box and that of the radiation is halved, then the total energy

(a) halves (b) doubles

(c) quadruples (d) remains the same

21. The displacement of a particle in a periodic motion is given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$.

This displacement may be considered as the result of superposition of n independent harmonic oscillations. Here n is

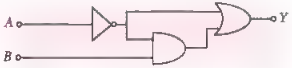
(a) 1 (b) 2 (c) 3 (d) 4

22. One mole of an ideal monoatomic gas is heated at a constant pressure from 0°C to 100°C. Then the change in the internal energy of the gas is (Given $R = 8.32$ J mol^{-1} K^{-1})

(a) 0.83×10^3 J (b) 4.6×10^3 J

(c) 2.08×10^3 J (d) 1.25×10^3 J

23. The output Y of the logic circuit given below is



(a) $\bar{A}+B$ (b) $\bar{A}$

(c) $\overline{(\bar{A}+B)}\cdot\bar{A}$ (d) $\overline{(\bar{A}+B)}\cdot A$

24. A whistle whose air column is open at both ends has a fundamental frequency of 5100 Hz. If the speed of sound in air is 340 m s^{-1}, the length of the whistle, in cm, is

(a) 5/3 (b) 10/3 (c) 5 (d) 20/3

25. To determine the coefficient of friction between a rough surface and a block, the surface is kept inclined at 45° and the block is released from rest. The block takes a time t in moving a distance d. The rough surface is then replaced by a smooth surface and the same experiment is repeated. The block now takes a time $t/2$ in moving down the same distance d. The coefficient of friction is

(a) 3/4 (b) 5/4 (c) 1/2 (d) $1/\sqrt{2}$

26. A smooth massless string passes over a smooth fixed pulley. Two masses m_1 and m_2 ($m_1 > m_2$) are tied at the two ends of the string. The masses are allowed to move under gravity starting from rest. The total external force acting on the two masses is

(a) $(m_1 + m_2)g$ (b) $\frac{(m_1 - m_2)^2}{m_1 + m_2}g$

(c) $(m_1 - m_2)g$ (d) $\frac{(m_1 + m_2)^2}{m_1 - m_2}g$

27. A wooden block is floating on water kept in a beaker. 40% of the block is above the water surface. Now the beaker is kept inside a lift that starts going upward with acceleration equal to $g/2$. The block will then

(a) sink

(b) float with 10% above the water surface

(c) float with 40% above the water surface

(d) float with 70% above the water surface

28. In which of the following phenomena, the heat waves travel along straight lines with the speed of light?

(a) Thermal conduction
(b) Forced convection
(c) Natural convection
(d) Thermal radiation

29. An artificial satellite moves in a circular orbit around the earth. Total energy of the satellite is given by E. The potential energy of the satellite is

(a) $-2E$ (b) $2E$
(c) $2E/3$ (d) $-2E/3$

30. A proton of mass m and charge q is moving in a plane with kinetic energy E. If there exists a uniform magnetic field B, perpendicular to the plane of the motion, the proton will move in a circular path of radius

(a) $\frac{2Em}{qB}$ (b) $\frac{\sqrt{2Em}}{qB}$ (c) $\frac{\sqrt{Em}}{2qB}$ (d) $\sqrt{\frac{2Eq}{mB}}$

31. A particle moves with constant acceleration along a straight line starting from rest. The percentage increase in its displacement during the 4^{th} second compared to that in the 3^{rd} second is

(a) 33% (b) 40% (c) 66% (d) 77%

32. A car is moving with a speed of 72 km hour^{-1} towards a roadside source that emits sound at a frequency of 850 Hz. The car driver listens to the sound while approaching the source and again while moving away from the source after crossing it. If the velocity of sound is 340 m s^{-1}, the difference of the two frequencies, the driver hears is

(a) 50 Hz (b) 85 Hz
(c) 100 Hz (d) 150 Hz

33. For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur?

(a) Isobar of original nucleus is produced.
(b) Isotope of the original nucleus is produced.
(c) Nuclei with higher atomic number than that of the original nucleus is produced.
(d) Nuclei with lower atomic number than that of the original nucleus is produced.

34. Same quantity of ice is filled in each of the two metal containers P and Q having the same size, shape and wall thickness but made of different materials. The containers are kept in identical surroundings. The ice in P melts completely in time t_1, whereas that in Q takes a time t_2. The ratio of thermal conductivities of the materials of P and Q is

(a) $t_2 : t_1$ (b) $t_1 : t_2$
(c) $t_1^2 : t_2^2$ (d) $t_2^2 : t_1^2$

35. When a particle executing SHM oscillates with a frequency υ, then the kinetic energy of the particle

(a) changes periodically with a frequency of υ.
(b) changes periodically with a frequency of 2υ.
(c) changes periodically with a frequency of $\upsilon/2$.
(d) remains constant.

36. A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now moved farther apart by pulling at them by means of insulating handles, then

(a) the energy stored in the capacitor decreases.
(b) the capacitance of the capacitor increases.
(c) the charge on the capacitor decreases.
(d) the voltage across the capacitor increases.

37. The ionization energy of hydrogen is 13.6 eV. The energy of the photon released when an electron jumps from the first excited state (n = 2) to the ground state of a hydrogen atom is

(a) 3.4 eV (b) 4.53 eV
(c) 10.2 eV (d) 13.6 eV

38. A uniform rod is suspended horizontally from its mid-point. A piece of metal whose weight is W is suspended at a distance l from the mid-point. Another weight W_1 is suspended on the other side at a distance l_1 from the mid-point to bring the rod to a horizontal position. When W is completely immersed in water, W_1 needs to be kept at a distance l_2 from the mid-point to get the rod back into horizontal position. The specific gravity of the metal piece is

(a) $\frac{W}{W_1}$ (b) $\frac{Wl_1}{Wl - W_1 l_2}$

(c) $\frac{l_1}{l_1 - l_2}$ (d) $\frac{l_1}{l_2}$

39. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be

(a) $2r\cos\left(\frac{\theta}{2}\right)$ (b) $2r\cot\left(\frac{\theta}{2}\right)$

(c) $2r\tan\left(\frac{\theta}{2}\right)$ (d) $2r\sin\left(\frac{\theta}{2}\right)$

40. A metal rod is fixed rigidly at two ends so as to prevent its thermal expansion. If L, α and Y respectively denote the length of the rod, coefficient of linear thermal expansion and Young's modulus of its material, then for an increase in temperature of the rod by ΔT, the longitudinal stress developed in the rod is

(a) inversely proportional to α

(b) inversely proportional to Y

(c) directly proportional to $\frac{\Delta T}{Y}$

(d) independent of L

41. The intermediate image formed by the objective of a compound microscope is

(a) real, inverted and magnified

(b) real, erect, and magnified

(c) virtual, erect and magnified

(d) virtual, inverted and magnified

42. Two coherent monochromatic beams of intensities I and $4I$ respectively are superposed. The maximum and minimum intensities in the resulting pattern are

(a) $5I$ and $3I$ (b) $9I$ and $3I$

(c) $4I$ and I (d) $9I$ and I

43. The energy of gamma (γ) ray photon is E_γ and that of an X-ray photon is E_x. If the visible light photon has an energy of E_v, then we can say that

(a) $E_x > E_\gamma > E_v$ (b) $E_\gamma > E_v > E_x$

(c) $E_\gamma > E_x > E_v$ (d) $E_x > E_v > E_\gamma$

44. A uniform solid spherical ball is rolling down a smooth inclined plane from a height h. The velocity attained by the ball when it reaches the bottom of the inclined plane is v. If the ball is now thrown vertically upwards with the same velocity v, the maximum height to which the ball will rise is

(a) $5h/8$ (b) $3h/5$ (c) $5h/7$ (d) $7h/9$

45. If the bandgap between valence band and conduction band in a material is 5.0 eV, then the material is

(a) semiconductor (b) good conductor

(c) superconductor (d) insulator

CATEGORY-II

Q.46 to Q.55 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark.

46. An object is placed 30 cm away from a convex lens of focal length 10 cm and a sharp image is formed on a screen. Now a concave lens is placed in contact with the convex lens. The screen now has to be moved by 45 cm to get a sharp image again. The magnitude of focal length of the concave lens is (in cm)

(a) 72 (b) 60 (c) 36 (d) 20

47. A 10 watt electric heater is used to heat a container filled with 0.5 kg of water. It is found that the temperature of water and the container rises by 3°K in 15 minutes. The container is then emptied, dried and filled with 2 kg of oil. The same heater now raises the temperature of container-oil system by 2°K in 20 minutes. Assuming that there is no heat loss in the process and the specific heat of water as 4200 J kg^{-1} K^{-1}, the specific heat of oil in the same unit is equal to

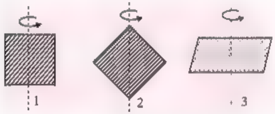
(a) 1.50×10^3 (b) 2.55×10^3

(c) 3.00×10^3 (d) 5.10×10^3

48. A solid uniform sphere resting on a rough horizontal plane is given a horizontal impulse directed through its centre so that it starts sliding with an initial velocity v_0. When it finally starts rolling without slipping the speed of its centre is

(a) $\frac{2}{7}v_0$ (b) $\frac{3}{7}v_0$ (c) $\frac{5}{7}v_0$ (d) $\frac{6}{7}v_0$

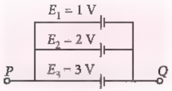
49. Three identical square plates rotate about the axes shown in the figure in such a way that their kinetic energies are equal. Each of the rotation axes passes through the centre of the square. Then the ratio of angular speeds $\omega_1 : \omega_2 : \omega_3$ is



(a) $1:1:1$ (b) $\sqrt{2}:\sqrt{2}:1$

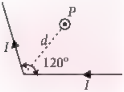
(c) $1:\sqrt{2}:1$ (d) $1:2:\sqrt{2}$

50. A circuit consists of three batteries of emf $E_1 = 1$ V, $E_2 = 2$ V and $E_3 = 3$ V and internal resistances 1 Ω, 2 Ω and 1 Ω respectively which are connected in parallel as shown in the figure. The potential difference between points P and Q is



(a) 1.0 V (b) 2.0 V (c) 2.2 V (d) 3.0 V

51. A long conducting wire carrying a current I is bent at 120° (see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is (μ_0 is the permeability of free space)



(a) $\frac{3\mu_0 I}{2\pi d}$ (b) $\frac{\mu_0 I}{2\pi d}$

(c) $\frac{\mu_0 I}{\sqrt{3}\pi d}$ (d) $\frac{\sqrt{3}\mu_0 I}{2\pi d}$

52. To determine the composition of a bimetallic alloy, a sample is first weighed in air and then in water. These weights are found to be w_1 and w_2 respectively. If the densities of the two constituent metals are ρ_1 and ρ_2 respectively, then the weight of the first metal in the sample is (where ρ_w is the density of water)

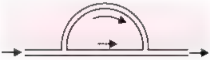
(a) $\frac{\rho_1}{\rho_w(\rho_2-\rho_1)}[w_1(\rho_2-\rho_w)-w_2\rho_2]$

(b) $\frac{\rho_1}{\rho_w(\rho_2+\rho_1)}[w_1(\rho_2-\rho_w)+w_2\rho_2]$

(c) $\frac{\rho_1}{\rho_w(\rho_2-\rho_1)}[w_1(\rho_2+\rho_w)-w_2\rho_1]$

(d) $\frac{\rho_1}{\rho_w(\rho_2-\rho_1)}[w_1(\rho_1-\rho_w)-w_2\rho_1]$

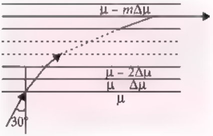
53. Sound waves are passing through two routes-one in straight path and the other along a semicircular path of radius r and are again combined into one pipe and superposed as shown in the figure. If the velocity of sound waves in the pipe is v, then frequencies of resultant waves of maximum amplitude will be integral multiples of



(a) $\frac{v}{r(\pi-2)}$ (b) $\frac{v}{r(\pi-1)}$

(c) $\frac{2v}{r(\pi-1)}$ (d) $\frac{v}{r(\pi+1)}$

54. A glass slab consists of thin uniform layers of progressively decreasing refractive indices RI (see figure) such that the RI of any layer is $\mu - m\Delta\mu$. Here μ and $\Delta\mu$ denote the RI of 0$^{\text{th}}$ layer and the difference in RI between any two consecutive layers, respectively. The integer m = 0, 1, 2, 3 .... denotes the numbers of the successive layers. A ray of light from the 0$^{\text{th}}$ layer enters the 1$^{\text{st}}$ layer at an angle of incidence of 30°. After undergoing the m^{th} refraction, the ray emerges parallel to the interface. If μ = 1.5 and $\Delta\mu$ = 0.015, the value of m is



(a) 20 (b) 30 (c) 40 (d) 50

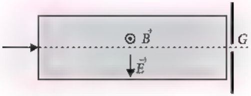
55. The de Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass is 0.5 MeV)

(a) 1 : 50 (b) 1 : 20
(c) 20 : 1 (d) 50 : 1

CATEGORY-III

Q.56 to Q.60 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question - irrespective of the number of correct options marked.

56. A stream of electrons and protons are directed towards a narrow slit in a screen (see figure). The intervening region has a uniform electric field $\vec{E}$ (vertically downwards) and a uniform magnetic field $\vec{B}$ (out of the plane of the figure) as shown. Then



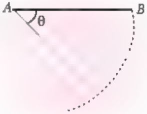
(a) electrons and protons with speed $\frac{|\vec{E}|}{|\vec{B}|}$ will pass through the slit.
(b) protons with speed $\frac{|\vec{E}|}{|\vec{B}|}$ will pass through the slit, electrons of the same speed will not.
(c) neither electrons nor protons will go through the slit irrespective of their speed.
(d) electrons will always be deflected upwards irrespective of their speed.

57. Half of the space between the plates of a parallel plate capacitor is filled with a dielectric material of dielectric constant K. The remaining half contains air as shown in the figure. The capacitor is now given a charge Q. Then

(a) electric field in the dielectric-filled region is higher than that in the air-filled region.
(b) on the two halves of the bottom plate the charge densities are unequal.
(c) charge on the half of the top plate above the air-filled part is $\frac{Q}{K+1}$.
(d) capacitance of the capacitor shown above is $(1+K)\frac{C_0}{2}$, where C_0 is the capacitance of the same capacitor with the dielectric removed.

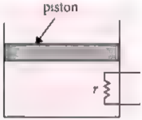
58. Find the correct statement(s) about photoelectric effect.
(a) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons.
(b) Einstein analysis gives a threshold frequency above which no electron can be emitted.
(c) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation.
(d) The maximum kinetic energy of electrons does not depend on the intensity of radiation

59. A thin rod AB is held horizontally so that it can freely rotate in a vertical plane about the end A as shown in the figure. The potential energy of the rod when it hangs vertically is taken to be zero. The end B of the rod is released from rest from a horizontal position. At the instant the rod makes an angle θ with the horizontal,



(a) the speed of end B is proportional to $\sqrt{\sin\theta}$.
(b) the potential energy is proportional to $(1 - \cos\theta)$.
(c) the angular acceleration is proportional to $\cos\theta$
(d) the torque about A remains the same as its initial value.

60. A heating element of resistance r is fitted inside an adiabatic cylinder which carries a frictionless piston of mass m and cross-section A as shown in diagram. The cylinder contains one mole of an ideal diatomic gas. The current flows through the element such that the temperature rises with time t as $\Delta T = \alpha t + \frac{1}{2}\beta t^2$ (α and β are constants), while pressure remains constant. The atmospheric pressure above the piston is P_0. Then



(a) the rate of increase in internal energy is $\frac{5}{2}R(\alpha + \beta t)$.

(b) the current flowing in the element is $\sqrt{\frac{5}{2r}R(\alpha+\beta t)}$.

(c) the piston moves upwards with constant acceleration.

(d) the piston moves upwards with constant speed.

SOLUTIONS

1. **(c)**: According to Stokes' law, the sphere encounters an opposing force F (or viscous force) when it falls through a viscous liquid of viscosity η with a velocity v and it is given by

$$F = 6\pi\eta av$$

Note : You can also see from the options, only option (c) has the dimensions of force.

2. **(a)** : Let a cricket ball be thrown with velocity u at an angle θ with the horizontal.

As per question,

$$h_1 = u\sin\theta t_1 - \frac{1}{2}gt_1^2$$

or $$u\sin\theta t_1 = h_1 + \frac{1}{2}gt_1^2 \quad \text{... (i)}$$

and $$h_2 = u\sin\theta t_2 - \frac{1}{2}gt_2^2$$

or $$u\sin\theta t_2 = h_2 + \frac{1}{2}gt_2^2 \quad \text{... (ii)}$$

Divide eqn. (i) by eqn. (ii), we get

$$\frac{t_1}{t_2} = \frac{h_1 + \frac{1}{2}gt_1^2}{h_2 + \frac{1}{2}gt_2^2}$$

$$h_2t_1 + \frac{1}{2}gt_2^2 t_1 = h_1t_2 + \frac{1}{2}gt_1^2t_2$$

$$h_1t_2 - h_2t_1 = \frac{1}{2}g(t_1t_2^2 - t_1^2t_2) \quad \text{... (iii)}$$

Time of flight, $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2}{g}\left[\frac{h_1 + \frac{1}{2}gt_1^2}{t_1}\right] \quad \text{(Using (i))}$$

$$= \frac{2}{g}\frac{h_1}{t_1} + t_1 = \frac{h_1}{t_1}\left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1 \quad \text{(Using (iii))}$$

$$= \frac{h_1t_2^2 - h_1t_1t_2}{h_1t_2 - h_2t_1} + t_1$$

$$= \frac{h_1t_2^2 - h_1t_1t_2 + h_1t_1t_2 - h_2t_1^2}{h_1t_2 - h_2t_1} = \frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$$

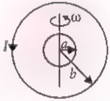
3. **(c)**: [Moment of a force] = [Torque]
= $[ML^2T^{-2}]$

[Moment of inertia] = $[ML^2T^0]$

4. **(c)**: The situation is shown in the figure.

The magnetic field at the common centre due to the current in larger loop is

$$B_l = \frac{\mu_0}{4\pi}\frac{2\pi I}{b} = \frac{\mu_0 I}{2b}$$



where subscript l refers to larger loop.

The magnetic flux linked with the smaller loop at time t is

$$\phi_s = B_l A_s \cos\omega t$$

where subscript s refers to smaller loop.

$$\phi_s = \frac{\mu_0 I}{2b}\pi a^2 \cos\omega t$$

The emf induced in the smaller loop is

$$\varepsilon_s = -\frac{d\phi_s}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I\pi a^2}{2b}\cos\omega t\right)$$

$$= \frac{\mu_0 I \pi a^2}{2b}\omega \sin \omega t$$

5. **(d)** : Here, radius of the orbit, $r = 0.05$ nm $= 0.05 \times 10^{-9}$ m

Frequency, $\upsilon = 10^{16}$ rev/s

The electron moving in a circular orbit behaves as a current loop.

The current due to orbital motion of electron is

$I = e\upsilon$

Area of the loop, $A = \pi r^2$

$\therefore$ Magnetic moment of the loop, $M = IA = e\upsilon\pi r^2$

$M = 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (0.05 \times 10^{-9})^2$
$= 0.0126 \times 10^{-21}$ A m^2 $= 1.26 \times 10^{-23}$ A m^2

6. **(c)**

7. **(b)** : In the active region I_B is in μA, I_C is in mA and V_{CE} is in volts.

8. **(a)**: According to Bohr's quantisation condition

Angular momentum, $L = n\frac{h}{2\pi}$

The force acting on a charge q moving with velocity v in a magnetic field B is

$F = qvB\sin\theta$

$\therefore [qB] = \frac{[F]}{[v]}$

$\therefore$ The dimensions of the quantity $\frac{nh}{2\pi qB}$ is

$$\left[\frac{nh}{2\pi qB}\right] = \frac{[L]}{[F]/[v]} = \frac{[L][v]}{[F]}$$

$$= \frac{[ML^2T^{-1}][LT^{-1}]}{[MLT^{-2}]} = [L^2] = [\text{Area}]$$

9. **(b)** : 

As the diode is ideal, it offers zero resistance when forward biased and offers infinite resistance when reverse biased.

When $V_i = 2$ V, the diode gets reverse biased and offers infinite resistance. Hence, no current flows in it.

i.e. $I_{initial} = 0$ A

When $V_i = 6$ V, the diode gets forward biased and offers zero resistance.

$\therefore I_{final} = \frac{6\text{ V} - 3\text{ V}}{150\ \Omega} = \frac{3}{150}\text{ A} = 0.02\text{ A}$

$\therefore$ Change in current, $\Delta I = I_{final} - I_{initial}$
$= 0.02$ A $- 0$ A $= 0.02$ A $= 20$ mA

10. **(a)** : Here, galvanometer resistance, $G = 10\ \Omega$

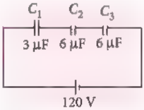
Current for full scale deflection, $I_g = 0.01$ A

To convert the galvanometer into a voltmeter of range 120 V, a high resistance R is connected in series with it such that

$V = I_g(G + R)$

$120 = 0.01(10 + R)$

or $R = \frac{120}{0.01} - 10 = 12000 - 10 = 11990\ \Omega$

11. **(d)** : 

In the given circuit, C_1, C_2 and C_3 are connected in series. The equivalent capacitance of these capacitors is

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$$

or $C_s = \frac{3}{2}$ μF

Let Q be charge on each capacitor.

$\therefore Q = (120\text{ V})\left(\frac{3}{2}\,\mu\text{F}\right) = 180\ \mu\text{C}$

Potential difference across C_1 (= 3 μF) is

$$V_1 = \frac{Q}{C_1} = \frac{180\ \mu\text{C}}{3\ \mu\text{F}} = 60\text{ V}$$

12. **(a)** : Here, $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

$\vec{C} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{X} = \alpha\vec{A} + \beta\vec{B} = \alpha(\hat{i} + \hat{j} - 2\hat{k}) + \beta(\hat{i} - \hat{j} + \hat{k})$

$= (\alpha + \beta)\hat{i} + (\alpha - \beta)\hat{j} + (\beta - 2\alpha)\hat{k}$

As $\vec{X} \perp \vec{C}$, so their dot product is zero.

i.e., $\vec{X} \cdot \vec{C} = 0$

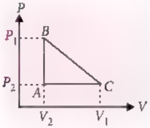
$((\alpha + \beta)\hat{i} + (\alpha - \beta)\hat{j} + (\beta - 2\alpha)\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$

$2\alpha + 2\beta - 3\alpha + 3\beta + 4\beta - 8\alpha = 0$

$9\alpha + 9\beta = 0$

$9\alpha - 9\beta \quad \text{or} \quad \frac{\alpha}{\beta} = 1 \quad \text{or} \quad \alpha : \beta = 1 : 1$

13. (a) :



For a cyclic process, $\Delta U = 0$

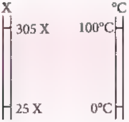
$\therefore$ According to first law of thermodynamics, the net heat absorbed by the gas is

$$\Delta Q = \text{Work done, } \Delta W$$

$$= \text{Area of } \Delta ABC$$

$$= \frac{1}{2}(P_1 - P_2)(V_1 - V_2)$$

14. (d) :



On the X scale, the temperature difference between the boiling and freezing points is

$305\ X - 25\ X = 280\ X$

On the °C scale, it is $100° - 0° = 100°C$

Thus, a temperature difference of 280 X is equal to a temperature difference of 100°C.

$\therefore$ For a 1°C change in °C scale, there is a corresponding 2.8 X change in X scale.

Specific heat capacity of water = 4200 J kg^{-1} $°C^{-1}$

Specific heat capacity of water in this new scale is

$$= \frac{4200}{2.8} \text{ J kg}^{-1} \text{ X}^{-1} = 1500 \text{ J kg}^{-1} \text{ X}^{-1}$$

$$= 1.5 \times 10^3 \text{ J kg}^{-1} \text{ X}^{-1}$$

15. (a) : Here, intensity of magnetisation,

$I = 5.0 \times 10^4$ A m^{-1}

Magnetic length, $L = 12$ cm $= 12 \times 10^{-2}$ m

Area of cross-section, $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$

Intensity of magnetisation,

$$I = \frac{\text{Magnetic moment } (M)}{\text{Volume } (V)}$$

or $\quad M = IV = IAL$

$= (5.0 \times 10^4 \text{ A m}^{-1})(1 \times 10^{-4} \text{ m}^2)(12 \times 10^{-2} \text{ m})$

$= 60 \times 10^{-2} \text{ A m}^2 = 0.6 \text{ A m}^2$

16. (a) : Here,

$\vec{r}_A = a(\hat{i} + 2\hat{j} + 3\hat{k}),\ \vec{r}_B = a(\hat{i} - 2\hat{j} + 6\hat{k})$

Displacement vector $\vec{r}$ from point A to B is

$\vec{r} = \vec{r}_B - \vec{r}_A$

$$= a(\hat{i} - 2\hat{j} + 6\hat{k}) - a(\hat{i} + 2\hat{j} + 3\hat{k}) = a(-4\hat{j} + 3\hat{k})$$

Electric field due to an infinite plane sheet of uniform surface charge density σ is

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

where $\hat{n}$ is a unit vector normal to the plane.

Here, $\hat{n} = \hat{k}$.

$$\therefore \quad \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{k}$$

Work done in moving a charge q from A to B is

$$W = \vec{F} \cdot \vec{r} = q\vec{E} \cdot \vec{r} \qquad (\because \vec{F} = q\vec{E})$$

$$= q\left(\frac{\sigma}{2\varepsilon_0}\hat{k}\right) \cdot a(-4\hat{j} + 3\hat{k}) = \frac{3\sigma a q}{2\varepsilon_0}$$

17. (d) : From lens displacement method,

$$f_{max} = \frac{d}{4}$$

18. (a) : When one cell is wrongly connected in a series of n cells each of emf E and internal resistance r, it will reduce the effective emf by $2E$ *i.e.* the effective emf $= nE - 2E$ whereas there is no effect on the total resistance of the cells.

Here, $n = 4$

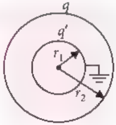
$\therefore$ Effective emf $= 4E - 2E = 2E$

Total internal resistance of the cells $= 4r$

The current in the external circuit is

$$I = \frac{2E}{4r + R}$$

19. (c):



Since the inner shell is grounded, therefore, its potential is zero. Let q' be charge on the inner shell. Then

$$V_{inner} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q'}{r_1}+\frac{q}{r_2}\right)=0$$

or $\frac{q'}{r_1}+\frac{q}{r_2}=0$ or $q'=-\frac{r_1}{r_2}q$

20. (a)

21. (c): $y=4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$

$y = 2(1 + \cos t)\sin(1000t)$ ($\because 2\cos^2\theta = 1 + \cos 2\theta$)

$= 2\sin(1000t) + 2\sin(1000t)\cos t$

$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$

($\because 2\sin A\cos B = \sin(A + B) + \sin(A - B)$)

So the given expression is the resultant of three independent harmonic oscillations.

22. (d) : Here, $R = 8.32$ J mol^{-1} K^{-1}

$\Delta T = 100°C - 0°C = 100°C$, $n = 1$

At constant pressure,

$\Delta Q = nC_P\Delta T$

and $\Delta W = nR\Delta T$

According to first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\Delta U = \Delta Q - \Delta W = nC_P\Delta T - nR\Delta T$

$= n\Delta T(C_P - R)$

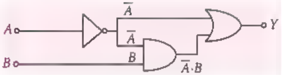
$= nC_V\Delta T$ ($\because C_P - C_V = R$)

For monoatomic gas, $C_V = \frac{3}{2}R$

$\therefore \Delta U = 1\times\frac{3}{2}\times 8.32\times 100 = 12.5\times 10^2$ J

$= 1.25 \times 10^3$ J

23. (b) :



The output Y of the logic circuit is

$Y=\bar{A}+\bar{A}\cdot B = \bar{A}(1+B)=\bar{A}\cdot 1=\bar{A}$

24. (b) : Let L be the length of the whistle.

Fundamental frequency, $\upsilon=\frac{v}{2L}$

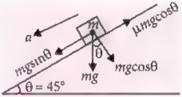
where v is the speed of sound in air.

or $L=\frac{v}{2\upsilon}$

Substituting the given values , we get

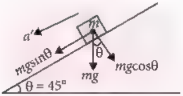
$$L=\frac{340}{2\times 5100}=\frac{1}{30}\text{ m}=\frac{10}{3}\text{ cm}$$

25. (a) :



The acceleration of the block down the rough inclined plane is

$a = g\sin\theta - \mu g\cos\theta = g(\sin\theta - \mu\cos\theta)$ .. (i)



The acceleration of the block down the smooth inclined plane is

$a' = g\sin\theta$ ... (ii)

As the block slides the distance d in each case,

$\therefore d=\frac{1}{2}at^2=\frac{1}{2}a't'^2$

($\because u = 0$ as block starts from rest)

$\therefore \frac{a}{a'}=\frac{t'^2}{t^2}$

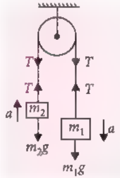
$\frac{g(\sin\theta-\mu\cos\theta)}{g\sin\theta}=\frac{t'^2}{t^2}$ (Using (i) and (ii))

$1-\mu\frac{\cos\theta}{\sin\theta}=\frac{t'^2}{t^2}$ or $1-\mu\cot\theta=\frac{t'^2}{t^2}$

Here, $\theta = 45°$, $t'=\frac{t}{2}$

$\therefore 1-\mu\cot 45°=\frac{1}{4}$ or $\mu=1-\frac{1}{4}=\frac{3}{4}$

26. (b) :



Let a be the common acceleration of the system and T be the tension in the string.
The equations of motion of two masses are

$$m_1g - T = m_1a \quad \text{... (i)}$$

$$T - m_2g = m_2a \quad \text{... (ii)}$$

Adding eqn. (i) and eqn. (ii), we get

$$a = \frac{(m_1 - m_2)}{m_1 + m_2}g \quad \text{... (iii)}$$

Acceleration of centre of mass of the system is

$$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2} = \frac{m_1a - m_2a}{m_1 + m_2}$$

($\because a_1$ and a_2 are equal in magnitude but opposite in direction)

$$a_{CM} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

(Using (iii))

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

The total external force acting on the two masses is

$$F_{ext} = (m_1 + m_2)a_{CM} = (m_1 + m_2)\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

$$= \frac{(m_1 - m_2)^2}{m_1 + m_2}g$$

27. (c)

28. (d) : Thermal radiation travel in straight lines with the speed of light.

29. (b) : Potential energy of the satellite
= 2(Total energy of the satellite)
= $2E$

30. (b) : As the proton moves in a circular path of radius r in the perpendicular magnetic field B

$$\therefore \frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

Kinetic energy of the proton, $E = \frac{1}{2}mv^2$

$$\text{or} \quad 2Em = m^2v^2 \quad \text{or} \quad mv = \sqrt{2mE}$$

$$\therefore r = \frac{\sqrt{2mE}}{qB}$$

31. (b) : Let a be constant acceleration of the particle.
Its displacement in n^{th} second is

$$D_{n\text{th}} = u + \frac{1}{2}a(2n-1)$$

Since the particle starts from rest, therefore $u = 0$

$$\therefore D_{n\text{th}} = \frac{1}{2}a(2n-1)$$

$$\therefore D_{3\text{rd}} = \frac{1}{2}a(2\times3-1) = \frac{5}{2}a$$

$$D_{4\text{th}} = \frac{1}{2}a(2\times4-1) = \frac{7}{2}a$$

% increase in its displacement

$$= \frac{D_{4\text{th}} - D_{3\text{rd}}}{D_{3\text{rd}}}\times100 = \frac{\frac{7}{2}a - \frac{5}{2}a}{\frac{5}{2}a}\times100$$

$$= \frac{a}{\frac{5}{2}a}\times100 = \frac{2}{5}\times100 = 40\%$$

32. (c): Here, velocity of the car,

$$v_{car} = 72 \text{ km hour}^{-1} = 72\times\frac{5}{18} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$$

Velocity of sound, $v = 340$ m s^{-1}
Frequency of source, $\upsilon = 850$ Hz
While approaching the source (stationary), the frequency heard by the driver is

$$\upsilon' = \upsilon\left(\frac{v + v_{car}}{v}\right)$$

While moving away from the source, the frequency heard by the driver is

$$\upsilon'' = \upsilon\left(\frac{v - v_{car}}{v}\right)$$

Difference in frequencies,

$$\Delta\upsilon = \upsilon' - \upsilon''$$

$$= \upsilon\left(\frac{v + v_{car}}{v}\right) - \upsilon\left(\frac{v - v_{car}}{v}\right) = \frac{2\upsilon v_{car}}{v}$$

$$= \frac{2\times850\times20}{340} = 100 \text{ Hz}$$

33. (b) : In α decay the mass number A and atomic number Z decrease by 4 and 2 respectively.
In β decay the mass number A remains the same but atomic number Z increases by 1.

34. (a) : Let K_1 and K_2 be the thermal conductivities of containers P and Q respectively.
As the same quantity of ice melts in P and Q,

the quantity of heat flows in P and Q must be same.

$$\therefore \quad Q = \frac{K_1 A(T_1 - T_2)t_1}{x} = \frac{K_2 A(T_1 - T_2)t_2}{x}$$

$$K_1 t_1 = K_2 t_2 \quad \text{or} \quad \frac{K_1}{K_2} = \frac{t_2}{t_1}$$

35. **(b)** : In SHM, kinetic energy varies periodically with double the frequency of SHM *i.e.* 2υ.

36. **(d)** : Charging battery is removed.
$\therefore$ Q = constant.
Distance between the plates is increased.

$\therefore$ C decreases. $\left(\because C = \frac{\varepsilon_0 A}{d}\right)$

Now, $V = \frac{Q}{C}$, Q is constant and C is decreasing.

$\therefore$ V increases.

$U = \frac{1}{2}\frac{Q^2}{C}$ again Q is constant and C is decreasing.

$\therefore$ U increases.

37. **(c)**: Energy of the electron in n^{th} state of hydrogen atom is

$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

For ground state, $n = 1$

$$\therefore \quad E_1 = -\frac{13.6}{1^2}\text{ eV}$$

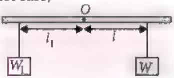
For first excited state, $n = 2$

$$\therefore \quad E_2 = -\frac{13.6}{2^2}\text{ eV}$$

The energy of the photon emitted for the electron transition from $n = 2$ to $n = 1$ is

$$\Delta E = E_2 - E_1 = (-13.6\text{ eV})\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2\text{ eV}$$

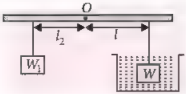
38. **(c)**: In first case,



Let O be midpoint of the rod.
For rotational equilibrium of the rod,
Taking moments about O, we get

$$W_1 l_1 = Wl \quad \text{... (i)}$$

In second case,



When metal piece W is immersed in water, it experiences a buoyant force.

$\therefore$ Apparent weight = Weight – Buoyant force

$W_{app} = W - F_B = V\rho_{metal}\,g - V\rho_{water}\,g$

[where V is the volume of the metal piece]

$$W_{app} = V\rho_{metal}\,g\left[1 - \frac{\rho_{water}}{\rho_{metal}}\right]$$

$$= W\left[1 - \frac{\rho_{water}}{\rho_{metal}}\right] \quad \text{... (ii)}$$

Again, for the rotational equilibrium of the rod,
Taking moments about O, we get

$W_1 l_2 = W_{app}\, l$

$$= Wl\left[1 - \frac{\rho_{water}}{\rho_{metal}}\right] \quad \text{(Using (ii))}$$

$$W_1 l_2 = W_1 l_1\left[1 - \frac{\rho_{water}}{\rho_{metal}}\right] \quad \text{(Using (i))}$$

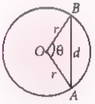
$$\frac{l_2}{l_1} = \left[1 - \frac{\rho_{water}}{\rho_{metal}}\right]$$

$$\frac{\rho_{water}}{\rho_{metal}} = 1 - \frac{l_2}{l_1} = \frac{l_1 - l_2}{l_1}$$

$\therefore$ Specific gravity of the metal piece

$$= \frac{\rho_{metal}}{\rho_{water}} = \frac{l_1}{l_1 - l_2}$$

39. **(d)** :



According to cosine formula

$$\cos\theta = \frac{r^2 + r^2 - d^2}{2r^2}$$

$2r^2\cos\theta = r^2 + r^2 - d^2$
$d^2 = 2r^2 - 2r^2\cos\theta = 2r^2[1 - \cos\theta]$

$$= 2r^2\left[2\sin^2\frac{\theta}{2}\right] \qquad \left[\because \cos\theta = 1 - 2\sin^2\frac{\theta}{2}\right]$$

or $\quad d = 2r\sin\left(\frac{\theta}{2}\right)$

$\therefore$ Linear displacement, $AB = d = 2r\sin\left(\frac{\theta}{2}\right)$

40. (d) : Strain $= \frac{\Delta L}{L}$

As $\Delta L = \alpha L \Delta T$

$\therefore \frac{\Delta L}{L} = \alpha\Delta T$

As $Y = \frac{\text{Stress}}{\text{Strain}}$

$\therefore$ Stress $= Y \times$ Strain $= Y\alpha\Delta T$

41. (a) : The intermediate image formed by the objective of a compound microscope is real, inverted and magnified.

42. (d) : $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

Here, $I_1 = I$ and $I_2 = 4I$

$\therefore \quad I_{max} = I + 4I + 2\sqrt{(I)(4I)} = 5I + 4I = 9I$

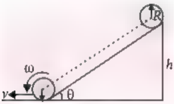
and $I_{min} = I + 4I - 2\sqrt{(I)(4I)} = 5I - 4I = I$

43. (c): Energy of a photon, $E = h\upsilon$

As $\upsilon_\gamma > \upsilon_x > \upsilon_v$

$\therefore \quad E_\gamma > E_x > E_v$

44. (c): Let m be mass and R be the radius of the ball.



According to law of conservation of mechanical energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$$

(For solid sphere, $I = \frac{2}{5}mR^2$)

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 \qquad (\because v = R\omega)$$

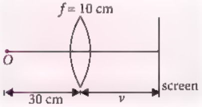
$$mgh = \frac{7}{10}mv^2 \quad \text{or} \quad v = \sqrt{\frac{10}{7}gh}$$

Let h' be maximum height attained by the ball when it is thrown vertically upwards with velocity $v\left(= \sqrt{\frac{10}{7}gh}\right)$.

$$\therefore \quad h' = \frac{v^2}{2g} = \frac{\left(\sqrt{\frac{10}{7}gh}\right)^2}{2g} = \frac{10}{14}h = \frac{5}{7}h$$

45. (d) : The band gap of 5.0 eV corresponds to that of an insulator.

46. (d) :



According to lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Here, $u = -30$ cm, $f = 10$ cm

$$\therefore \quad \frac{1}{v} - \frac{1}{-30} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15} \quad \text{or} \quad v = 15 \text{ cm}$$

Now a concave lens of focal length f' is placed in contact with convex lens. In order to obtain a sharp image again the screen has to be moved by 45 cm.

$\therefore$ New image distance, $v' = v + 45$ cm
$= 15$ cm $+ 45$ cm
$= 60$ cm

Let F be focal length of the combination.

Again using lens formula

$$\therefore \quad \frac{1}{F} = \frac{1}{v'} - \frac{1}{u}$$

$$\frac{1}{F} = \frac{1}{60} - \frac{1}{-30} = \frac{1}{60} + \frac{1}{30} = \frac{1}{20}$$

or $\quad F = 20$ cm

As $\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$

$$\therefore \quad \frac{1}{20} = \frac{1}{10} + \frac{1}{f'} \quad \text{or} \quad \frac{1}{f'} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20}$$

or $\quad f' = -20$ cm

$|f'| = 20$ cm

47. (b) : Here, power of heater, $P = 10$ W

Mass of water, $m_w = 0.5$ kg

Mass of oil, $m_o = 2$ kg

Specific heat of water, $s_w = 4200$ J kg^{-1} K^{-1}

In case of container-water system

Energy supplied by the heater to the system in 15 min = 10 × 15 × 60 = 9000 J

$\therefore \quad m_w s_w \Delta T_w + m_c s_c \Delta T_c = 9000$

(where subscripts w and c refer to water and container respectively.)

$$0.5 \times 4200 \times 3 + m_c \times s_c \times 3 = 9000$$

$$6300 + 3m_c s_c = 9000$$

$$3m_c s_c = 2700$$

$$m_c s_c = 900 \qquad \text{... (i)}$$

In case of container-oil system

Energy supplied by the same heater to the system in 20 min = 10 × 20 × 60 = 12000 J

$\therefore \quad m_o s_o \Delta T_o + m_c s_c \Delta T_c = 12000$

(where subscript o refers to oil)

$$2 \times s_o \times 2 + 900 \times 2 = 12000 \qquad \text{(Using (i))}$$

$4s_o = 10200$

$s_o = 2550$ J kg^{-1} K^{-1} = 2.55×10^3 J kg^{-1} K^{-1}

Hence, the specific heat of oil is

2.55×10^3 J kg^{-1} K^{-1}.

48. (c):

Let M and R be mass and radius of the solid sphere respectively.

Applying law of conservation of angular momentum about the point of contact, we get

$$Mv_0R = MvR + \frac{2}{5}MR^2\omega$$

$$\left(\because \text{ For solid sphere, } I = \frac{2}{5}MR^2\right)$$

$$= MvR + \frac{2}{5}MR^2\left(\frac{v}{R}\right) = \frac{7}{5}MvR \quad (\because v = R\omega)$$

$$v_0 = \frac{7}{5}v \quad \text{or} \quad v = \frac{5}{7}v_0$$

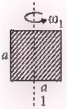
49. (b) : Kinetic energy of rotation, $K_R = \frac{1}{2}I\omega^2$

or $\omega^2 = \dfrac{2K_R}{I}$

Since K_R remains the same for all three configurations,

$\therefore \quad \omega \propto \dfrac{1}{\sqrt{I}}$

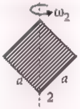
Let m be the mass and a be side of each square plate



In figure 1, moment of inertia about the given axis,

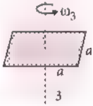
$$I_1 = \frac{1}{12}ma^2$$

In figure 2, moment of inertia about the given axis,



$$I_2 = \frac{1}{12}ma^2$$

In figure 3, moment of inertia about the given axis



$$I_3 = \frac{1}{6}ma^2$$

$\therefore \quad I_1 : I_2 : I_3 = 1 : 1 : 2$

As $\omega \propto \dfrac{1}{\sqrt{I}}$

$$\therefore \quad \omega_1 : \omega_2 : \omega_3 = 1 : 1 : \frac{1}{\sqrt{2}} = \sqrt{2} : \sqrt{2} : 1$$

50. (b) : Here, $E_1 = 1$ V, $E_2 = 2$ V, $E_3 = 3$ V

$r_1 = 1\ \Omega$, $r_2 = 2\ \Omega$, $r_3 = 1\ \Omega$

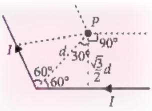
The effective emf of the circuit is

$$E_{eff} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} = \frac{\frac{1}{1} + \frac{2}{2} + \frac{3}{1}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{1}} = \frac{\frac{10}{2}}{\frac{5}{2}} = \frac{10}{5}$$

$= 2$ V

$\therefore$ Potential difference between points P and Q $= E_{eff} = 2$ V

51. (d) :



Magnetic field at P due to current through the bent wire is

$$B = 2\left[\frac{\mu_0}{4\pi}\frac{I}{\left(\frac{\sqrt{3}}{2}d\right)}(\sin 90° + \sin 30°)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi}\frac{I}{\left(\frac{\sqrt{3}}{2}d\right)}\left(1 + \frac{1}{2}\right)\right] = \frac{\sqrt{3}\mu_0 I}{2\pi d}$$

52. (a): Let x be weight of the first metal in the alloy.

$\therefore$ Weight of second metal in the alloy $= w_1 - x$

Let V_1 and V_2 be volumes of two constituent metals in the alloy.

$$\therefore \quad V_1 = \frac{x}{g\rho_1} \quad \text{and} \quad V_2 = \frac{w_1 - x}{g\rho_2}$$

$\therefore$ Total volume of the alloy

$$V = V_1 + V_2 = \left(\frac{x}{g\rho_1} + \frac{w_1 - x}{g\rho_2}\right) \quad \text{... (i)}$$

As the weight of the alloy in air and water are w_1 and w_2 respectively.

$\therefore$ Loss in weight = Weight of water displaced

$w_1 - w_2 = V\rho_w g = (V_1 + V_2)\rho_w g$

$$w_1 - w_2 = \left(\frac{x}{g\rho_1} + \frac{w_1 - x}{g\rho_2}\right)\rho_w g \quad \text{(Using(i))}$$

$$= \left(\frac{x}{\rho_1} + \frac{w_1 - x}{\rho_2}\right)\rho_w$$

$$w_1 - w_2 = \frac{x\rho_2\rho_w + w_1\rho_1\rho_w - x\rho_1\rho_w}{\rho_1\rho_2}$$

$x\rho_2\rho_w - x\rho_1\rho_w = w_1\rho_1\rho_2 - w_2\rho_1\rho_2 - w_1\rho_1\rho_w$

$x\rho_w[\rho_2 - \rho_1] - \rho_1[w_1(\rho_2 - \rho_w) - w_2\rho_2]$

$$x = \frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 - \rho_w) - w_2\rho_2]$$

53. (a) :

The path difference between two interfering waves is

$\Delta x = \pi r - 2r = r(\pi - 2)$

For maximum amplitude, $\Delta x = n\lambda$

$$r(\pi - 2) = n\lambda \quad \text{or} \quad \lambda = \frac{r(\pi - 2)}{n}$$

$$\text{Frequency, } \upsilon = \frac{v}{\lambda} = n\left[\frac{v}{r(\pi - 2)}\right]$$

54. (d) : By Snell's law, $\mu\sin i$ = constant

$\therefore \quad \mu\sin 30° = (\mu - m\Delta\mu)\sin 90°$

$\mu\sin 30° = \mu - m\Delta\mu$

$m\Delta\mu = \mu - \mu\sin 30°$

$$m = \frac{\mu(1 - \sin 30°)}{\Delta\mu}$$

Here, $\mu = 1.5$, $\Delta\mu = 0.015$

$$\therefore \quad m = \frac{1.5\left(1 - \frac{1}{2}\right)}{0.015} = \frac{0.75}{0.015} = 50$$

55. (c): de Broglie wavelength of an electron

$$\lambda_e = \frac{h}{\sqrt{2m_e K_e}}$$

where subscript e refers to electron.

$\therefore$ Kinetic energy of the electron,

$$K_e = \frac{h^2}{2m_e\lambda_e^2}$$

$$\text{Energy of a photon, } E_{ph} = \frac{hc}{\lambda_{ph}}$$

where subscript 'ph' refers to photon.

$$\therefore \quad \frac{E_{ph}}{K_e} = \frac{hc}{\lambda_{ph}} \times \frac{2m_e\lambda_e^2}{h^2} = \frac{2m_e\lambda_e^2 c}{h\lambda_{ph}}$$

As per question, $\lambda_e = \lambda_{ph}$

$$\therefore \quad \frac{E_{ph}}{K_e} = \frac{2m_e c\lambda_{ph}}{h} = \frac{2m_e c^2}{hc/\lambda_{ph}}$$

$$= \frac{2 \times 0.5 \times 10^6 \text{ eV}}{50 \times 10^3 \text{ eV}} = \frac{20}{1}$$

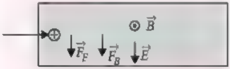
or $\quad E_{ph} : K_e = 20 : 1$

56. (c, d) : A charge q moving with velocity $\vec{v}$ in presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$ experiences a Lorentz force and is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = \vec{F}_E + \vec{F}_B$$

If the total force $\vec{F}$ on the charge is zero, the charge will move in the fields undeflected.

For proton



For electron

57. (b, c, d) : Let A be area of each plate and d is the distance between the plates.

The given capacitor is equivalent to two capacitors in parallel with capacitances

$$C_1 = \frac{K\varepsilon_0(A/2)}{d} = \frac{K\varepsilon_0 A}{2d} \quad \text{... (i)}$$

$$C_2 = \frac{\varepsilon_0(A/2)}{d} = \frac{\varepsilon_0 A}{2d} \quad \text{... (ii)}$$

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{K\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{2d}$$

$$= \frac{\varepsilon_0 A}{2d}(K+1) = \frac{C_0}{2}(K+1)$$

where $C_0 = \frac{\varepsilon_0 A}{d}$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{1} \quad \text{... (iii) (Using (i) and (ii))}$$

As surface charge density, $\sigma = \frac{\text{Charge}}{\text{Area}}$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{Q_1}{Q_2} = \frac{K}{1} \quad \text{... (iv)} \qquad \text{(Using (iii))}$$

Total charge, $Q = Q_1 + Q_2$ ... (v)

From (iii) and (v), we get

$$Q_1 = \frac{KQ}{K+1} \quad \text{and} \quad Q_2 = \frac{Q}{K+1}$$

Electric field in dielectric filled region,

$$E_1 = \frac{\sigma_1}{\varepsilon_0 K}$$

Electric field in air-filled region,

$$E_2 = \frac{\sigma_2}{\varepsilon_0}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} \times \frac{1}{K} = K \times \frac{1}{K} = 1 \qquad \text{(Using (iv))}$$

58. (a, d) : The photoelectric emission is an instantaneous process without any apparent time lag.

Below threshold frequency no electron can be emitted.

According to Einstein's photoelectric equation,

$$K_{max} = h\upsilon - \phi_0$$

where υ is the frequency of incident radiation and ϕ_0 is the work function.

The maximum kinetic energy of electrons is independent of the intensity of radiation.

59. (a, c)

60. (a, c): Increase in internal energy,

$$dU = nC_V\,dT$$

$\therefore$ Rate of increase in internal energy

$$\frac{dU}{dt} = nC_V \frac{dT}{dt}$$

Here, $n = 1$, $dT = \alpha t + \frac{1}{2}\beta t^2$

$$\therefore \quad \frac{dT}{dt} = \alpha + \beta t \quad \text{... (i)}$$

For diatomic gas, $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$

$$\therefore \quad \frac{dU}{dt} = 1 \times \frac{5}{2}R \times (\alpha + \beta t) = \frac{5}{2}R(\alpha + \beta t)$$

Heat supplied by the heating element to the gas at constant pressure,

$$dQ = nC_P\,dT$$

Rate of heat supplied,

$$\frac{dQ}{dt} = nC_P \frac{dT}{dt} = 1 \times \frac{7}{2}R \times (\alpha + \beta t) \quad \text{(Using(i))}$$

$$= \frac{7}{2}R(\alpha + \beta t)$$

Let I be current flowing in the element.

$$\therefore \quad I^2 r = \frac{7}{2}R(\alpha + \beta t)$$

$$I^2 = \frac{7}{2r}R(\alpha + \beta t)$$

$$I = \sqrt{\frac{7}{2r}R(\alpha + \beta t)}$$

According to ideal gas equation,

$$PV = nRT$$

At constant pressure,

$$PdV = nRdT$$

$$PdV = nR\left(\alpha t + \frac{1}{2}\beta t^2\right)$$

Work done, $W = PdV = nR\left(\alpha t + \frac{1}{2}\beta t^2\right)$ ... (ii)

Let F be force exerted by the gas on the piston of area A.

$$\therefore \quad P = \frac{F}{A} \quad \text{or} \quad F = PA$$

Let the piston moved upwards by a distance x.

$$\therefore \quad W = Fx$$

$$\text{or} \quad x = \frac{W}{F} = \frac{nR}{PA}\left(\alpha t + \frac{1}{2}\beta t^2\right)$$

Velocity, $v = \frac{dx}{dt} = \frac{nR}{PA}(\alpha + \beta t)$

Acceleration, $a = \frac{dv}{dt} = \frac{nR}{PA}\beta$

CBSE BOARD 2015

CHAPTERWISE PRACTICE PAPER

SERIES-1

Electrostatics

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) There are 30 questions in total. Questions Nos. 1 to 8 are very short answer type questions and carry one mark each.

(iii) Questions Nos. 9 to 18 carry two marks each. Questions Nos. 19 to 27 carry three marks each and questions Nos. 28 to 30 carry five marks each.

(iv) One of the questions carrying three marks weightage is value based question.

(v) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions of five marks each weightage. You have to attempt only one of the choices in such questions.

(vi) Use of calculators is not permitted. However, you may use log tables if necessary.

(vii) You may use the following values of physical constants wherever necessary :

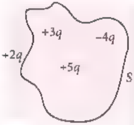
$$c = 3 \times 10^8 \text{ m s}^{-1},\ h = 6.63 \times 10^{-34} \text{ J s},\ e = 1.6 \times 10^{-19} \text{ C},$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1},\ \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2},\ m_e = 9.1 \times 10^{-31} \text{ kg}$$

1. What is the expression of capacitance of a parallel plate capacitor with a conducting slab of thickness t ?

2. Can we give as much charge to a capacitor as we wish?

3. Four point charges $+2q$, $+3q$, $-4q$ and $+5q$ are as shown in the figure. What is the electric flux due to this configuration through the surface S ?



4. What is the SI unit of surface integral of electric field?

5. Does the electric potential rise or fall along an electric line of force?

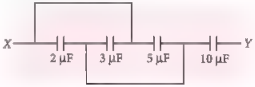
6. What is the electric field in the cavity, if a conductor having a cavity is charged? Does the result depend on the shape and size of cavity of the conductor?

7. What is an equipotential surface? Give an example.

8. Two charged particles system separated by distance 2 m is placed in air and experience a force 10 N. If this system is placed in a medium of dielectric constant 5 to experience the same magnitude of the force then what would be the separation between them?

9. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.7×10^{-22} C m^{-2}. Find the electric field strength E
   (a) in the outer region of the first plate
   (b) in the outer region of the second plate and
   (c) between the plates.

10. Four capacitors are connected as shown in the figure. Find the equivalent capacitance between the points X and Y.



11. A spherical Gaussian surface encloses a charge of 17.7×10^{-8} C.
(a) Find the electric flux passing through the Gaussian surface.
(b) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface ?

12. An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 N C^{-1}. Find the magnitude of the torque acting on the dipole.

13. Find the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away.

Hence, obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

14. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge q.

15. What is the electrostatic shielding ? Write its two application.

16. What is electric polarization? Define polarization density.

OR

Two identical spheres A and B, each having a charge of 6×10^{-8}C, are separated by 60 cm. A third uncharged sphere C of the same size is brought in contact with sphere A, then brought in contact with sphere B and finally removed from both. What is the new force of repulsion between A and B?

17. How will you combine three capacitors, each of 3 μF, so that their equivalent capacitance is
(i) 9 μF, (ii) 1 μF, (iii) 4.5 μF ?

18. What is the potential gradient at a distance of 10^{-12} m from the centre of the platinum nucleus? What is the potential gradient at the surface of the nucleus? Atomic number of platinum is 78 and radius of platinum nucleus is 5×10^{-15} m.

19. What is meant by the statement that the electric field of a point charge has spherical symmetry whereas that of an electric dipole is cylindrically symmetric?

20. While watching a science show on atmosphere, Pooja find that on the average an electric field of the Earth is about 100 N C^{-1} directed vertically downward.
(a) Why is this field not felt by a person standing on the surface of the Earth?
(b) What is the total charge on the Earth's surface?
(c) Is this charge large as compared to the total charge of all the electrons in copper coin of mass 3.10 g? (For Cu, $Z = 29$; Take radius of Earth = 6.37 10^6 m)

21. (a) Determine the electrostatic potential energy of a system consisting of two charges 7 μc and –2 μc (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.
(b) How much work is required to separate the two charges infinitely away from each other?
(c) Suppose that the some system of charges is now placed in an external electric field $E - A(1/r^2)$; $A - 9 \times 10^5$ C m^{-2}. What would be electrostatic energy of the configuration be?

22. (a) Two fixed point charges $+4e$ and $+e$ units are separated by a distance a. Where should a third point charge be placed for it to be in equilibrium?
(b) Why is it easier to charge a balloon on a dry day than on a humid day?

23. Derive an expression for the electric field at any point on the axial line of an electric dipole.

24. The capacitance of a capacitor with vacuum is 0.088 μF. When a dielectric is introduced, the capacitance of the same capacitor becomes 0.330 μF.
(a) Find the dielectric constant of the medium.
(b) If the electric field strength in the absence of dielectric is 5000 V m^{-1}, find the field strength in the presence of the dielectric.

OR

Twenty seven drops of water of the same size are equally and similarly charged. They are then united to form a bigger drop. How will capacitance and potential change?

25. (a) Depict the equipotential surfaces for a system of two identical positive point charges placed at a distance d apart.

(b) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r}_1$ and $\vec{r}_2$ respectively in the presence of external electric field $\vec{E}$.

26. State Gauss' theorem of electrostatic. Using this theorem, derive an expression for the electric field intensity due to an infinite plane sheet of charge density σ C m^{-2}.

27. Explain the underlying principle of working of a parallel plate capacitor.

If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expressions for

(i) the electric field at points between the two plates.

(ii) the potential difference between the plates

(iii) the capacitance of the capacitor so formed.

28. Explain the principle on which Van de Graaff generator operates. Draw a labelled schematic sketch and write briefly its working.

OR

(a) Find the expression of the electric field at equatorial position due to a dipole. Assume dipole length is much smaller than the equatorial position.

(b) If the dipole is placed in a uniform electric field, find the expression of torque acting on it.

29. (a) Find the expression of electric potential at a point due to point charge.

(b) Show that work done in moving a charge particle around a closed path in electric field is zero.

(c) Write three properties of equipotential surfaces.

OR

Find the expression of electric field intensity due to a hollow charged shell of radius R at a point which is at distance r if

(i) $r < R$ (ii) $r = R$ (iii) $r > R$

Also, draw the graph showing the variation of electric field with distance.

30. Find the expression of electric potential due to an electric dipole at

(i) an axial position. (ii) equatorial position.

(iii) any point due to short dipole.

OR

(a) What is electrostatic potential energy? Derive an expression of potential energy for a system of three point charges situated at the corners of a triangle.

(b) If n capacitors of capacitance C_1, C_2, C_3, ... C_n are connected in series, find the expression of equivalent capacitance.

SOLUTIONS

1. Capacitance of a parallel plate capacitor with a conducting slab, $C = \dfrac{\varepsilon_0 A}{d-t}$

where A = Area of the plate

d = separation between plates

t = thickness of conducting slab

ε_0 = permittivity of free space.

2. No, the maximum charge that can be given to capacitor could be determined by the capacity of the condenser.

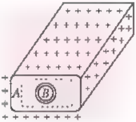
3. According to Gauss's law, total electric flux over the closed surface S,

$$\phi = \frac{Q}{\varepsilon_0} = \frac{+3q - 4q + 5q}{\varepsilon_0} \Rightarrow \phi = \frac{4q}{\varepsilon_0}$$

4. The SI unit of surface integral of electric field is N m^2 C^{-1} or volt metre.

5. The electric potential falls along an electric line of force because electric line of force is always directed from higher to lower potential.

6. When a conductor is charged, charge resides on its outer surface, In figure, for Gaussian surface A in the conductor or B in the cavity,



$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0} = 0 \quad \Rightarrow \quad E = 0$$

That is electric field vanishes in a conductor or in a cavity. This is independent of shape and size of conductor and cavity.

7. Any surface which has same electric potential at every point is called an equipotential surface. The surface of a charged conductor is an equipotential surface.

8. According to Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad ...(i)$$

or $$F_m = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r_m^2} \quad ...(ii)$$

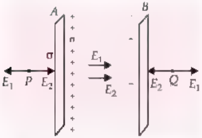
According to question,

$F = F_m$

$$\therefore \frac{1}{K}\frac{1}{r_m^2} = \frac{1}{r^2} \quad \therefore K r_m^2 = r^2$$

or $$r_m = \frac{r}{\sqrt{K}} = \frac{2}{\sqrt{5}} \text{ m}$$

9. The electric field due to each surface charge $= \frac{\sigma}{2\varepsilon_0}$.



Given $\sigma = 17.7 \times 10^{-22}$ C m^{-2}

(a) The electric field in the outer region of first plate (point P).

$$= E_2 - E_1 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(b) The electric field in outer origin of second plate (point Q).

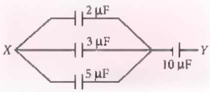
$$= E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(c) The electric field between the plates

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$= \frac{17.7\times10^{-22}}{8.85\times10^{-11}} = 2\times10^{-11}\text{ N C}^{-1}.$$

10. Equivalent circuit is as shown below



Capacitance in parallel,

$C_p = (2 + 3 + 5)$ μF $= 10$ μF.

Capacitance in series,

$$C_s = \frac{10\times10}{10+10} = \frac{100}{20} = 5\,\mu\text{F}$$

$\therefore$ Equivalent capacitance, $C_{eq} = 5$ μF

11. (a) Electric flux passing through the Gaussian surface

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{17.7\times10^{-8}}{8.85\times10^{-12}} = 2\times10^4\text{ N m}^2\text{ C}^{-1}$$

(b) If the radius of the Gaussian surface is doubled, there is no change in flux.

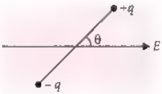
12. Here, $p = 4\times10^{-9}$ C m,

$\theta = 30°$,

$E = 5\times10^4$ N C^{-1}

Torque,

$\tau = pE\sin\theta$



$$= 4\times10^{-9}\times5\times10^4\times\sin 30°$$

$$= 4\times5\times10^{-5}\times\frac{1}{2} = 10^{-4}\text{ N m}$$

13. Potential at a point P,

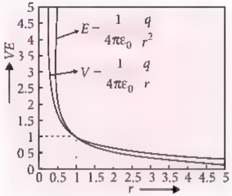
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 4\times10^4\text{ V}$$

Work done in bringing the charge,

$$W = qV = 2\times10^{-9}\times4\times10^4$$
$$= 8\times10^{-5}\text{ J}$$

As electrostatic field is conservative, work done does not depend upon path along which the charge is brought

14.



15. The phenomenon of making a region free from any external electric field is called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

Applications of electrostatic shielding :

(i) In a thunderstorm accompanied by lightning, it is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lighting.

(ii) Sensitive components of electronic devices

are protected from external electric disturbances by placing metal shields around them.

16. When a non-polar dielectric is placed in an electric field, the negative and positive charges shift in opposite directions. As a result the dielectric develops induced dipole moment. This phenomenon is called electric polarization.
The induced dipole moment per unit volume is called polarization density.

OR

Given, $q_A = q_B = 6 \times 10^{-8}$ C
When uncharged sphere C is brought in contact with sphere A, then the charge on A or C is

$$q'_A = \frac{\text{Charge on } A + \text{Charge on } C}{2}$$

$$= \frac{(6\times10^{-8}+0)}{2} = 3\times10^{-8}\text{ C}$$

When charged sphere C is brought in contact with charged sphere B, then the charge on sphere B or C is

$$q'_B = \frac{(6\times10^{-8}+3\times10^{-8})}{2} = 4.5\times10^{-8}\text{ C}$$

New force between spheres A and B is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q'_A q'_B}{r^2} = \frac{9\times10^9\times3\times10^{-8}\times4.5\times10^{-8}}{(0.6)^2}$$

$$= \frac{9\times3\times4.5\times10^{-7}}{0.36} = 3.4\times10^{-5}\text{ N}$$

17. (i) In parallel combination, equivalent capacitance, $C = C_1 + C_2 + C_3$

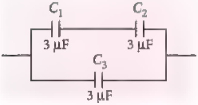
$$= 3 + 3 + 3 = 9\ \mu\text{F}$$

(ii) In series combination, equivalent capacitance,

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$$

or $\quad C = 1\ \mu$F

(iii) Two capacitors should be connected in series and the third in parallel with the series combination as shown in figure.



$$C = \frac{C_1C_2}{C_1+C_2}+C_3 = \frac{3\times3}{6}+3$$

$$= 1.5 + 3 = 4.5\ \mu\text{F}$$

18. Here, $q = Ze = 78 \times 1.6 \times 10^{-19}$ C
Potential gradient at a point is numerically equal to electric field at that point,

i.e., $\dfrac{dV}{dr} = E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

At $r = 10^{-12}$ m,

$$\frac{dV}{dr} = E = \frac{9\times10^9\times78\times1.6\times10^{-19}}{(10^{-12})^2}$$

$$= 1.123 \times 10^{17}\text{ V m}^{-1}$$

At $r = 5 \times 10^{-15}$ m

$$\frac{dV}{dr} = E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times78\times1.6\times10^{-19}}{(5\times10^{-15})^2}$$

$$= 4.5 \times 10^{21}\text{ V m}^{-1}$$

19. The electric field due to a point charge q at distance r from it is given by $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

Clearly, the magnitude of field $\vec{E}$ will be same at all points on the surface of a sphere of radius r drawn around the point charge and does not depend on the direction of $\vec{r}$. Hence the field due to a point charge is spherically symmetric.
Electric field at distance r on the equatorial line of an electric dipole of dipole moment p is given by $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{(r^2+a^2)^{3/2}}$

The electric field E is same at all points which lie on a cylinder of radius r with its axis on the dipole axis and the field pattern looks same in all planes passing through the dipole axis. We say that the electric field of an electric dipole is cylindrically symmetric.

20. (a) The human body is a conductor and as such it becomes an equipotential surface while standing on the Earth in its electric field.
(b) Total charge on the Earth's surface,

$$Q = \sigma A = (\varepsilon_0 E)(4\pi R^2) = 4\pi\varepsilon_0 ER^2$$

$$= 4\times3.14\times 8.85\times10^{-12}\times 100\times(6.37\times10^6)^2$$

$$= 4.51\times10^5\text{ C}$$

This charge is negative as $\vec{E}$ is directed vertically downward.

(c) Number of copper atoms in 3.10 g of copper, *i.e.*,

$$N = 6.02\times10^{23}\times\left(\frac{3.10}{63.5}\right) = 2.94\times10^{22}$$

Number of electrons in the copper coin,
$n = 29$ N
$= 29\times2.94\times10^{22} = 8.53\times10^{23}$

Total charge in the copper coin,
$Q' = 8.53\times10^{23}\times1.6\times10^{-19}$
$= 1.37\times10^{5}$ C

$$\therefore\ \frac{Q}{Q'} = \frac{4.51\times10^{5}}{1.37\times10^{5}} = 3.3$$

$\Rightarrow Q = 3.3Q'$

Thus, total charge on the surface of the Earth is only 3.3 times the total charge of all the electrons in a single copper coin.

21. (a) Given $q_1 = +7\ \mu$C, $q_2 = -2\ \mu$C and $r_{12} = 9 + 9 = 18$ cm

So electrostatic potential energy of the system is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}} = 9\times10^{9}\ \frac{7\times10^{-6}\times-2\times10^{-6}}{18\times10^{-2}}$$

$U = -0.7$ J

(b) $W = \Delta U = U_\infty - U = 0 - (-0.7)$
or $W = +0.7$ J

(c) $E = A\times\dfrac{1}{r^2}$ or $\dfrac{-dV}{dr} = A\times\dfrac{1}{r^2}$

or $dV = -A\times\dfrac{1}{r^2}\cdot dr$

Integrating it on both sides, we get

$$\int_{V_\infty}^{V} dV = -A\int_{\infty}^{r}\frac{1}{r^2}dr = -A\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$[V]_{V_\infty}^{V} = A\left[\frac{1}{r}\right]_{\infty}^{r}$$

or $V - V_\infty = A\left[\dfrac{1}{r} - \dfrac{1}{\infty}\right] = A\left[\dfrac{1}{r} - 0\right]$

or $V - 0 = \dfrac{A}{r}$ or $V = \dfrac{A}{r}$ ...(i)

The mutual interaction energy of the two charges remains unchanged. In addition, there is the energy of interaction of the two charges with the external field and hence the net electrostatic energy is

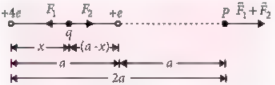
$$U = q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

or $U = \dfrac{q_1A}{r_1} + \dfrac{q_2A}{r_2} + \dfrac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$

or $U = \dfrac{7\times10^{-6}\times9\times10^{5}}{0.09\text{ m}} + \dfrac{-2\times10^{-6}\times9\times10^{5}}{0.09} - 0.7$

or $U = 70 - 20 - 0.7$ or $U = 49.3$ J

22. (a)



Let the third point charge (say q) be at a distance x from $+4e$ as shown in figure. Since the distance between $+4e$ and $+e$ is a, distance between q and $+e = (a - x)$

For q to be in equilibrium,
force between q and $+4e$, = force between q and $+e$, *i.e.*, $F_1 = F_2$

or $k_e\dfrac{q\times4e}{x^2} = k_e\dfrac{q\times e}{(a-x)^2}$

or $\dfrac{4}{x^2} = \dfrac{1}{(a-x)^2}$ or $\dfrac{2}{x} = \pm\dfrac{1}{(a-x)}$

$\dfrac{2}{x} = \dfrac{1}{(a-x)}$, $x = 2a - 2x$

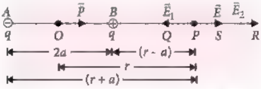
or $x = \dfrac{2a}{3}$

Thus, q should be at a distance of $2a/3$, from $+4e$ charge.

Note: If we take $\dfrac{2}{x} = -\dfrac{1}{(a-x)}$, $x = 2a$, which is not feasible. This is because the point P for equilibrium lies on the right side of $+e$ at a distance $2a$ from $+4e$ (or a from $+e$). In this case, F_1 and F_2 are in the same direction and their resultant $(\vec{F}_1 + \vec{F}_2)$ cannot be zero and as such equilibrium is not possible.

(b) The moisture allows some of the charge to leave the balloon.

23. A line passing through the positive and negative charges of the dipole is called the axial line of the electric dipole.



Consider a point P on the axial line of a dipole (situated in a vacuum) of length $2a$ at a distance r from the midpoint O as shown in figure.

Electric field intensity at P due to $-q$ charge at A, *i.e.*

$$E_1 = k_e \frac{q}{AP^2} = k_e \frac{q}{(r+a)^2}$$

It is presented in magnitude and direction by $\overline{PQ}$, *i.e.*, $\vec{E}_1$. Electric field intensity at P due to $+q$ charge at B, *i.e.*

$$E_2 = k_e \frac{q}{BP^2} = k_e \frac{q}{(r-a)^2}$$

It is represented in magnitude and direction by $\overline{PR}$, *i.e.* $\vec{E}_2$.

Let $\vec{E}$ be the resultant electric intensity at P

According to the principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Since $\vec{E}_1$ and $\vec{E}_2$ act in opposite directions, the resultant electric intensity $(\vec{E})$ at P due to the dipole is represented by $\overline{PS}$.

Clearly, as $|\vec{E}_2| > |\vec{E}_1|$,

$$E = E_2 - E_1 = k_e\left[\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2}\right]$$

$$= k_e q\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right] = k_e q\frac{4ra}{(r^2-a^2)^2}$$

or $\quad E = k_e\frac{2\times 2qa\times r}{(r^2-a^2)^2} = k_e\frac{2pr}{(r^2-a^2)^2}$

Since $\vec{E}$ and $\vec{p}$ are in the same direction,

$$\vec{E} = k_e\frac{2\vec{p}r}{(r^2-a^2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}r}{(r^2-a^2)^2}$$

24. (a) Here, $C_0 = 0.088\ \mu F$, $C = 0.330\ \mu F$

Dielectric constant, $K_r = \frac{C}{C_0} = \frac{0.330}{0.088} = 3.75$

(b) Here, $E_0 = 5000\ \text{V m}^{-1}$

As $\quad K_r = \frac{E_0}{E} = \frac{E_0}{E_0 - E_i}$,

$K_r E_0 - K_r E_i = E_0$

or $\quad (K_r - 1)E_0 = K_r E_i$

or $\quad E_i = \frac{(K_r-1)}{K_r}E_0$

$$= \frac{(3.75-1)}{3.75}\times(5000\ \text{V m}^{-1})$$

$$E_i = 3667\ \text{V m}^{-1}$$

OR

Here, number of small drops, $n = 27$

Let r = radius of each small drop

R = radius of the bigger drop.

Since volume of bigger drop = volume of 27 small drops,

$$\left(\frac{4\pi}{3}\right)R^3 = 27\left(\frac{4\pi}{3}\right)r^3$$

or $\quad R^3 = 27r^3 = (3r)^3$ or $R = 3r$

As capacitance of a sphere is directly proportional to its radius and radius of the bigger drop is three times the radius of a small drop,

capacitance of bigger drop = 3 times capacitance of a small drop.

If q is the charge on each small drop, then potential of each small drop,

$$V_S = k_e\frac{q}{r} \quad \text{...(i)}$$

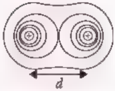
Total charge on the bigger drop, $Q = 27q$

Potential of the bigger drop,

$$V_B = k_e\frac{Q}{R} = k_e\frac{27q}{3r} = 9V_S \quad \text{(Using (i))}$$

Hence, the potential of the bigger drop is 9 times the potential of a small drop.

25. (a) Equipotential surfaces for a system of two identical positive point charges placed a distance d apart as shown in figure below.



(b) Work done in bringing the charge q_1 from infinity to $\vec{r}_1$ against the external electric field

$$W_1 = q_1 V(r_1)$$

Work done in bringing the charge q_2 from infinity to $\vec{r}_2$ against the field

$$W_2 = q_2 V(r_2)$$

Work done on q_2 against the field due to q_1

$$W_3 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

Potential energy of the system

$$= W_1 + W_2 + W_3$$

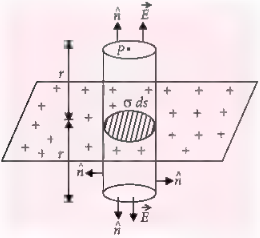
$$= q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

26. Gauss's theorem : Surface integral of electric field or electric flux over a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

i.e., $\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$ or $\phi_E = \frac{q}{\varepsilon_0}$

Given : An infinite plane sheet of charge of surface charge density σ C m^{-2}.

Assume a cylindrical Gaussian surface passing normally through the sheet. Let the area of cross-section of the cylinder be *ds*.



Total charge enclosed by the surface of the cylinder, = σds

Total flux through Gaussian surface,

$\phi_E = 2Eds$ ...(i)

Flux through curved surface of the cylinder is zero, because at each point on curved surface electric field vector is perpendicular to the area vector.

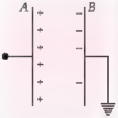
According to Gauss's theorem $\phi_E = \frac{\sigma ds}{\varepsilon_0}$ ...(ii)

From (i) and (ii)

$$2Eds = \frac{\sigma ds}{\varepsilon_0} \quad \therefore \quad E = \frac{\sigma}{2\varepsilon_0}$$

Electric field intensity at a point is independent of its distance from the charge sheet.

27. Principle of capacitor : A capacitor works on the principle that the capacitance of a conductor increases appreciably when an earthed conductor is brought near it.



A parallel plate capacitor consists of two metallic plates *A* and *B* separated by a distance *d*. Let plate *A* be given a positive charge *q*.

When an earthed plate *B* is brought near it, a negative charge (–*q*) is induced on the inner side and a positive charge (+*q*) is induced on the outer side of plate *B*; due to being earthed the charge +*q* induced as *B* is transferred to earth, so due to presence of (–*q*) charge on plate *B*, the potential of plate *A* is appreciably reduced, so capacitance of conductor $A, C = \frac{q}{V}$ is appreciably increased; thus a parallel capacitor has two metallic plates having charges +*q* and –*q*.

(i) Electric field at points between the two plates $E = \frac{\sigma}{\varepsilon_0}$

(ii) Potential difference between the plates is

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

(iii) Capacitance of the capacitor so formed is

$$C = \frac{\varepsilon_0 A}{d}$$

28. *Refer point 11 Page No.17 (MTG Excel in Physics).*

OR

(a) *Refer point 1.4 (4) page No. 6 (MTG Excel in Physics).*

(b) *Refer point 1.4 (5) Page No. 6, (MTG Excel in Physics).*

29. (a) *Refer point 1.5 (5) Page No. 8 (MTG Excel in physics).*

(b) *Refer point 1.5 (8) Page No. 8 (MTG Excel in physics).*

(c) *Refer point 1.5 (11) Page No. 9 (MTG Excel in physics).*

OR

Refer point 1.8 (4) Page No. 13 (MTG Excel in physics).

30. *Refer point 1.5 (10) Page No. 9 (MTG Excel in Physics).*

OR

(a) *Refer point 1.5 (12), Page No. 10 (MTG Excel in Physics).*

(b) *Refer point 1.11 (7) Page No. 16 (MTG Excel in Physics).*

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?
Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

Q1. How will you change a digital circuit to a analogue one, and a analogue to a digital one?
– Priyadarsi (W.B.)

Ans. Analogue and digital concept in the simplest form can be explained like this. When one measures the blood pressure of a person, the height of mercury which is reflecting the pressure in the veins fluctuating like heart beats is in the analogue form. If the readings are taken quantitatively, it is digital.
But the answer, how to convert digital to analogue and analogue to digital in instruments is this. Every counter and spectrometer and other instruments give the results in both. But how to convert is taught to you when you are working in electronic engineering. These will not be given by the instrument makers because each one has a method of his own. They will not share the secrets. You can learn the principles in colleges but the actual way it is done will be learnt when you join the establishments.

Q2. What is the path of the moving leaf falling from a tree and what is its velocity?
- Mahesh Bhatt Pithoragarh (Uttarakhand)

Ans. One cannot predict the path of a falling leaf because it depends on the force due to the wind, gravitation and the variation of the shape.

This is the reason why in learning in laboratories, every force is defined both in direction and magnitude.

Even in the manufacturing of planes, the wind tunnels with a particular direction of wind under controlled conditions are used for understanding the effect of wind.

**Q3. Current can pass through a wet wood but not through a dry wood why?
Is there any role of water? If the wood is dipped in alcohol then what is the result?**
- Komal Mohanty Kazidhta (Cuttack)

Ans. Dry wood is an insulator. Wet wood conducts electricity.
If wood is dipped in alcohol, alcohol catches fire. Therefore the wood will burn faster.

Q4. What exactly would happen to the solar system if the Earth's gravitational force suddenly vanishes ?
Rahul Roy Chowdhury (W.B.)

Ans. The Earth has mass, just like every other solid object does. It is the Earth's mass that causes it to have gravity, and so in order to not have gravity the Earth would have to not have mass. But if the Earth didn't have mass, it wouldn't be there anymore!
If the Earth's gravity vanish then things not attached to Earth is any other way would fly off into space in a straight line that would takes them away from the surface of the Earth. The earth's atmosphere itself would also float off into space. The Earth itself would most likely break apart into chunks and float off into space, since it is held together by gravity only. So Earth as such will no more be a part of solar system if Earth's gravitational force vanishes.

Solution Senders of Physics Musing

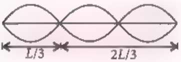
SET-10

1. Abhishek Prasad (Jamshedpur)
2. Alok Verma (Lucknow)
3. Sangeeta (Patna)
4. Debranjan Basu (W.B.)

PHYSICS MUSING

SOLUTION SET-10

1. **(a,d)** : Since, there is a node at a distance $\frac{L}{3}$ from one end,

(i) 

Minimum no. of nodes = 2

(ii) 

Next higher no. of nodes = 5

For case (i) $\lambda_1 = \frac{2L}{3}$,

therefore, fundamental frequency, $\upsilon_1 = \frac{v}{\lambda_1} = \frac{3v}{2L}$

where v is velocity of waves in the string

2. **(a, b, d)** : Given: $\frac{I_1}{I_2} = 4.84, D = 96$ cm

Let $I_1 = 4.84\,a$, $I_2 = a$

$\therefore \quad O = \sqrt{I_1 I_2} = 2.2a$

(a) Required ratio $= \frac{2.2\,a}{a} = \frac{11}{5}$

(b) $\frac{V}{u} = \frac{11}{5}$ ... (i)

$v + u = 96$ ... (ii)

From equations (i) and (ii), we get,

$$V + \frac{5V}{11} = 96 \Rightarrow \frac{16V}{11} = 96$$

$\Rightarrow v = 66$ cm and $u = 30$ cm

Distance between two position of the lens $= v - u$

$= 66 - 30 = 36$ cm

(c) $\frac{1}{f} = \frac{1}{66} + \frac{1}{30} = \frac{30 + 66}{30 \times 66}$

$\Rightarrow f = \frac{30 \times 66}{96} = \frac{330}{16} = 20.625$ cm

(d) $u = 30$ cm

3. **(a, b, c, d)** : $\beta = \frac{\lambda}{2\mu\theta}$

4. **(a, c, d)** : Considering the two point masses and bar as system,

$P_c = 0$

$\therefore \quad V_c = 0.$

By conservation of angular momentum,

we have, $L_i = L_f$

$$2mVa + m \times 2V \times 2a = \left\{\frac{8m(6a)^2}{12} + 2ma^2 + m(2a)^2\right\}\omega$$

On solving, we get

$$\omega = \frac{V}{5a}$$

Now, $E = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 30ma^2 \times \omega^2 = \frac{3}{5}mV^2$

5. **(b, c)** : Work done by the spring on block

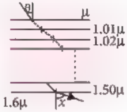
= loss in spring potential energy

$= \frac{1}{2}ka^2 - \frac{1}{2}kb^2 = \frac{1}{2}k(a^2 - b^2)$

This is also work done against friction $= \mu mg(a + b)$

$$\therefore \quad \mu = \frac{\frac{1}{2}k(a^2 - b^2)}{mg\,(a + b)} = \frac{k}{2mg}(a - b)$$

6. **(a)**

7. **(b)** : 

According to Snell's law,

$\mu\sin\theta = 1.6\mu\sin x$

$$\therefore \quad \sin x = \frac{5}{8}\sin\theta$$

8. **(b)** : $Q = \int_{T_0}^{\eta T_0} C\,dT$

$$= \int_{T_0}^{\eta T_0} \frac{a}{T}dT = a\ln\frac{\eta T_0}{T_0} = a\ln\eta$$

$$\Delta U = C_V \Delta T = \frac{R}{\gamma - 1}(\eta - 1)T_0 \quad \left(\because C_V = \frac{R}{\gamma - 1}\right)$$

$$\therefore \quad W = Q - \Delta U = a\ln\eta - \left[\frac{\eta - 1}{\gamma - 1}\right]RT_0$$

9. When outer surface is grounded charge '$-Q$' resides on the inner surface of sphere 'B'.
Now sphere 'A' is connected to earth, potential on its surface becomes zero.
Let the charge on the surface A becomes q

$$\frac{kq}{a}-\frac{kQ}{b}=0 \Rightarrow q=\frac{a}{b}Q$$

In this position energy stored

$$E_1=\frac{1}{8\pi\varepsilon_0 a}\left[\frac{a}{b}Q\right]^2+\frac{Q^2}{8\pi\varepsilon_0 b}+\frac{1}{4\pi\varepsilon_0 b}\left[\frac{a}{b}Q\right](-Q)$$

When 'S_3' is closed, total charge will appear on the outer surface of shell 'B'.
In this position energy stored

$$E_2=\frac{1}{8\pi\varepsilon_0 b}\left(\frac{a}{b}-1\right)^2 Q^2$$

Heat produced $= E_1 - E_2$

$$=\frac{Q^2\,a(b-a)}{8\pi\varepsilon_0 b^3}=1.8\text{ J}$$

10. Time taken by the pulse to reach from P to Q,

$$t_0=\frac{F_0}{k} \qquad \text{...(i)}$$

where $F_0 = 3$ N

$$\text{Now, } v=\sqrt{\frac{T}{m}} \Rightarrow \frac{dx}{dt}=\sqrt{\frac{F_0-kt}{m}}$$

$$\therefore \int_0^L dx=\frac{1}{\sqrt{m}}\int_0^{t_0}(F_0-kt)^{1/2}dt$$

$$L=\frac{1}{\sqrt{m}}\frac{2}{(-3k)}\left[(F_0-kt)^{3/2}\right]_0^{t_0}$$

$$=\frac{2}{-3k\sqrt{m}}(F_0-kt_0-F_0+k(0))^{3/2}$$

$$=\frac{2}{-3k\sqrt{m}}\left(\frac{-kF_0}{k}\right)^{3/2} \qquad \text{(Using (i))}$$

$$L=\frac{2}{3k\sqrt{m}}F_0^{3/2}$$

$$\therefore k=\frac{2}{3L}\sqrt{\frac{F_0^3}{m}}=\frac{2}{3\times 1}\sqrt{\frac{3\times 3\times 3}{3\times 10^{-2}}}$$

$$=\frac{2\times 3}{3\times 0.1}=20\text{ N s}^{-1}$$

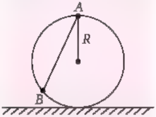
Thought Provoking Problems

Kinematics

By : Prof. Rajinder Singh Randhawa*

1. Two balls are thrown towards each other, ball A at $16\ m\ s^{-1}$ upward from the ground and ball B at $9\ m\ s^{-1}$ downward from a roof 30 m high, one second later. (a) Where and when do they meet? (b) What are their velocities on impact?

2. A circular wire frame is fixed in a vertical plane. A smooth wire is slightly stretched between points A and B. A bead slides from the highest point A, the highest point of the circle.

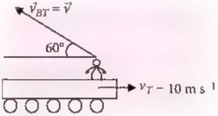


Determine (a) its velocity v when it arrives at B, and (b) show that time to slide along any similar chord is same.

3. A point A moves uniformly with a velocity v in such a way that direction of its velocity continually points at another point B, which in turn moves along a straight line with a uniform velocity $u (u < v)$. At the initial moment u and v are separated by a distance l. How soon will the points meet?

4. A gun is fired from a moving platform and the ranges of the shot are observed to be R and S when the platform is moving forward and backward respectively with velocity V. Prove that the elevation of the gun is $\tan^{-1}\left[\dfrac{g(R-S)^2}{4V^2(R+S)}\right]$.

5. A boy is riding on a flat car of a train moving with velocity $10\ m\ s^{-1}$. The boy throws



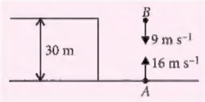
a ball which according to him makes an angle 60° with the horizontal and in line with the track. An observer on the ground observes the ball to rise vertically. How high does he see the ball rise?

6. A small sphere of mass m is released from rest in a large vessel filled with oil where it experiences a resistive force proportional to its speed, *i.e.*, $F_d = -kv$. (a) Find the law according to which the ball's speed varies. (b) After a certain time the sphere reaches a terminal speed, find it. (c) Time constant is the time it takes the sphere to reach 63.2% of its terminal speed, find it if $m = 2$ g and terminal speed is $5\ cm\ s^{-1}$. (d) Determine the time it takes the sphere to reach 90% of its terminal speed.

*Randhawa Institute of Physics, S.C.O. 208, First Fl., Sector-36D & S.C.O. 38, Second Fl., Sector-20C, Chandigarh

SOLUTIONS

1. (a) Let A is in motion for time t s, then B has been in motion for time $(t - 1)$ s.



$\therefore$ Distance travelled by ball A in t s,

$y_A = 16t - 4.9t^2$ ...(i)

Distance travelled by ball B in $(t - 1)$ s

$y_B = 9(t - 1) + 4.9(t - 1)^2$ ...(ii)

When they meet, $y_A + y_B = 30$

i.e. $16t - 4.9t^2 + 9(t - 1) + 4.9(t - 1)^2 = 30$

On solving we get $t = 2.24$ s

From equation (i), $y_A = 11.3$ m

and from equation (ii), $y_B = 18.7$ m

Therefore, the balls meet 2.24 s after ball A is thrown and at 11.3 m from the ground.

(b) $v_A = 16 + (-9.8)t$ ...(iii)

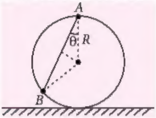
and $v_B = 9 + (9.8)(t - 1)$ ...(iv)

Now put $t = 2.24$ s in equation (iii) and equation (iv), we get

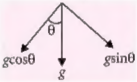
$v_A = -5.95$ m s^{-1}

$v_B = 21.15$ m s^{-1}

2. Length of the chord $AB = 2R\cos\theta$



Acceleration of bead along wire, $a = g\cos\theta$



(a) From, $v^2 - u^2 = 2as$

$v^2 = 0^2 + 2(g\cos\theta)(2R\cos\theta)$

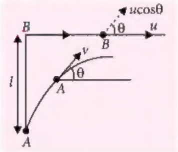
or $v = 2\sqrt{Rg}\cos\theta$

(b) From $v = u + at = 0 + at$ or $t = \dfrac{v}{a}$

$$\therefore \quad t = \frac{2\sqrt{Rg}\cos\theta}{g\cos\theta} = 2\sqrt{\frac{R}{g}}$$

which is independent of θ, thus time to slide along any chord is same.

3.



When A meets B in time T, separation is decreased by

$$\int_0^T (v - u\cos\theta)dt = l \quad \text{...(i)}$$

In time T distance moved by $B = uT$.

Since A catches B its displacement along x-direction must be same as that of B.

$$B\cdot\int_0^T v\cos\theta dt = uT \quad \text{...(ii)}$$

$$\text{or} \quad \int_0^T \cos\theta dt = \frac{uT}{v} \quad \text{...(iii)}$$

Put equation (iii) in equation (i), we get

$$vT - u\left(\frac{uT}{v}\right) = l \quad \text{or} \quad T = \frac{vl}{v^2 - u^2}$$

4. Consider u and v be the horizontal and vertical components of the velocity when the platform is at rest. When the platform is moving forward, the resultant velocity is $(u + V)$ and vertical velocity is v.

The range is given by

$$R = \frac{2(u + V)v}{g} \quad \text{...(i)}$$

When the platform is moving backward with velocity V, then range is

$$S = \frac{2(u - V)v}{g} \quad \text{...(ii)}$$

From equations (i) and (ii)

$$R + S = \frac{4uv}{g} \quad \text{and} \quad R - S = \frac{4Vv}{g}$$

$$\therefore \quad \frac{(R - S)^2}{(R + S)} = \frac{16V^2 v^2/g^2}{4uv/g} = \frac{4V^2 v}{ug}$$

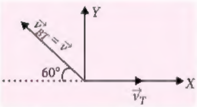
If α is the angle of projection, then $\tan\alpha = \dfrac{v}{u}$,

or $\dfrac{(R-S)^2}{(R+S)} = \dfrac{4V^2}{g}\tan\alpha \quad \therefore \quad \tan\alpha = \dfrac{g(R-S)^2}{4V^2(R+S)}$

or $\quad \alpha = \tan^{-1}\left[\dfrac{g(R-S)^2}{4V^2(R+S)}\right]$

5. The velocity of the ball with respect to the train

$\vec{v}_{BT} = \vec{v}$



Since the ground observer sees it to move vertically

$\vec{v}_{BT} = \vec{v}_B - \vec{v}_T \quad \text{or} \quad \vec{v}_B = \vec{v}_{BT} + \vec{v}_T$

Since the ball moves in the vertical direction its X-component of velocity w.r.t. ground must vanish.

$v_{BX} = -v\cos 60° + v_T = 0$ ...(i)

and Y-component of velocity of ball w.r.t. ground, $v_{BY} = v\sin 60°$

From (i), $v = \dfrac{v_T}{\cos 60°} = \dfrac{10 \text{ m s}^{-1}}{1/2} = 20 \text{ m s}^{-1}$

$\therefore \quad v_{BY} = 20\sin 60° = 20\dfrac{\sqrt{3}}{2} \text{ m s}^{-1}$

Maximum height reached by ball as seen by ground observer,

$$h = \frac{(v_{BY})^2}{2g} = \frac{\left(20\frac{\sqrt{3}}{2}\right)^2}{2\times 9.8}$$

$\therefore \quad h = 15.30$ m

6. (a) Force on the sphere = $mg - kv$, where k is a constant

$\therefore$ acceleration of the ball,

$\dfrac{dv}{dt} = g - \dfrac{k}{m}v$ ...(i)

or $\dfrac{-dv}{\frac{mg}{k} - v} = -\dfrac{k}{m}dt$ ...(ii)

Integrating both sides,

$\ln\left(\dfrac{mg}{k} - v\right) = -\dfrac{k}{m}t + C$ ...(iii)

If $v = 0$ at $t = 0$, equation (iii) gives

$C = \ln\dfrac{mg}{k}$

Put C in equation (iii), we get

$\ln\left\{\dfrac{mg/k - v}{mg/k}\right\} = -\dfrac{k}{m}t$

or $\quad v = \dfrac{mg}{k}(1 - e^{-t/\tau})$ ...(iv)

where $\tau = \dfrac{m}{k}$ is called time constant.

(b) When the particle reaches terminal speed, the acceleration of the particle becomes zero. Then particle continue to move at constant speed called terminal speed.

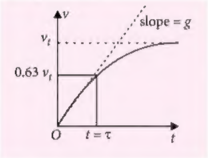
$mg = kv_t \quad \text{or} \quad v_t = \dfrac{mg}{k}$

(c) $k = \dfrac{mg}{v_t} = \dfrac{2\times 980}{5} = 392 \text{ g s}^{-1}$

Time constant, $\tau = \dfrac{m}{k} = \dfrac{2}{392}$

or $\quad \tau = 5.10 \times 10^{-3}$ s

(d) Speed of the particle as a function of time is given by equation (iv),



$v = v_t(1 - e^{-t/\tau})$

or $\quad 0.90v_t = v_t(1 - e^{-t/\tau})$

$1 - e^{-t/\tau} = 0.90 \Rightarrow e^{-t/\tau} = 0.10$

Taking log, we get $\dfrac{-t}{\tau} = \ln(0.10) = -2.30$

or $\quad t = 2.30\tau$

$\therefore \quad t = 2.30 \times 5.10 \times 10^{-3} = 11.73 \times 10^{-3}$ s

 Contd. from page no. 26

$$\lambda_e = \frac{12.3}{\sqrt{V}}\,\text{Å} = \frac{12.3}{\sqrt{100}}\,\text{Å} = \frac{12.3}{10}\,\text{Å} = 1.23\,\text{Å}$$

$(\because V = 100\text{ V})$

Now, wavelength of light, $\lambda = \lambda_e = 1.23$ Å

$d\theta = 9 \times 10^{-9}$ rad.

$\therefore$ Aperture of telescope,

$$D = \frac{1.22\lambda}{d\theta} = \frac{1.22\times 1.23\times 10^{-10}}{9\times 10^{-9}}\text{ m}$$

$$= 0.0167\text{ m} = 16.7\text{ mm}$$

25. (b) : Let I_0 be intensity of incident light, then the intensity of light emerging from the first polaroid,

$$I_1 = \frac{I_0}{2}$$

Initially, the two polaroids are crossed to each other *i.e.* $\theta_i = 90°$

Let the polaroid be rotated by angle θ, then the angle between polarising directions is $90° - \theta$

Now, intensity of light emerging from the second polaroid,

$$I_2 = I_1\cos^2(90° - \theta) = \frac{I_0}{2}\cos^2(90° - \theta)$$

Also, $I_2 = 25\%$ of $I_0 = \frac{I_0}{4}$

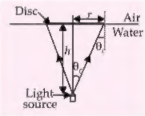
$$\therefore\ \frac{I_0}{4} = \frac{I_0}{2}\cos^2(90° - \theta)$$

$$\Rightarrow\ \cos^2(90° - \theta) = \frac{1}{2}$$

$$\text{or}\ \cos(90° - \theta) = \frac{1}{\sqrt{2}} = \cos 45°$$

$$\text{or}\ \theta = 90° - 45° = 45°$$

26. (a) : The figure shows incidence from water at critical angle θ_c for the limiting case.



Now, $\sin\theta_c = \frac{1}{\mu}$ so that

$$\tan\theta_c = \frac{1}{(\mu^2 - 1)^{1/2}}$$

From figure, $\tan\theta_c = \frac{r}{h}$

where r is the radius of the disc.

Therefore, diameter of the disc is

$$2r = 2h\tan\theta_c = \frac{2h}{(\mu^2 - 1)^{1/2}}$$

27. (c): According to Brewster's law, the light reflected from the top of glass slab gets polarised as shown in figure.

The light refracted into the glass slab and the light emerging from the glass slab is only partially polarised. Therefore, when a polaroid is held in the path of emergent light at P, and rotated about an axis passing through the centre and perpendicular to plane of polaroid, the intensity of light shall go through a minimum but not zero for two orientations of the polaroid.

28. (a) : When the final image is formed at least distance of distinct vision d, magnifying power of the telescope is

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{d}\right) = \frac{140}{5}\left(1 + \frac{5}{25}\right)$$

$$= 28[1 + 0.2] = 28 \times 1.2 = 33.6$$

29. (a) : Resolution of pinhole camera $= \frac{1.22\lambda}{d}$

Here d is the diameter of the pupil.

Also, Resolution $= \frac{\text{Separation of objects}}{\text{Distance of object}}$

$$\frac{1.22\lambda}{d} = \frac{1\text{ mm}}{R}$$

$$\Rightarrow\ \frac{1.22\times 500\times 10^{-9}}{3\times 10^{-3}} = \frac{1\times 10^{-3}}{R}$$

$$\Rightarrow\ R = 5\text{ m}$$

30. (a) : Here, angular resolution of human eye, $\phi = 5.8 \times 10^{-4}$ rad

The linear distance between two successive dots in a typical photoprinter is

$$l = \frac{2.54}{300}\text{ cm} = 0.84\times 10^{-2}\text{ cm.}$$

At a distance of z cm, the gap distance l will subtend an angle

$$\phi = \frac{l}{z_P}\quad \therefore\ z_P = \frac{l}{\phi} = \frac{0.84\times 10^{-2}\text{ cm}}{5.8\times 10^{-4}} = 14.5\text{ cm}$$